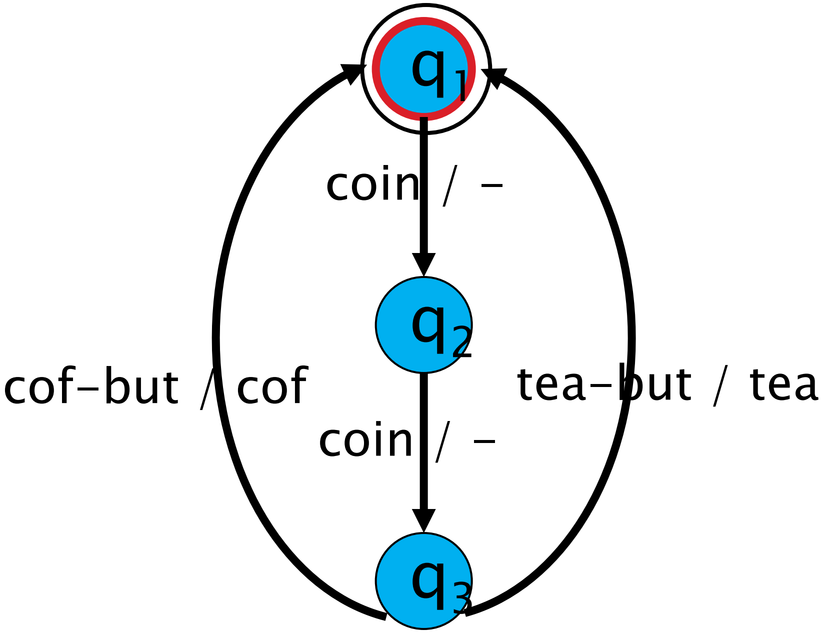
Model checking timed transition systems: timed automata

Lecture 5

Finite State Machine (Mealy)



condition		effect	
current state	input	output	next state
q_1	coin	-	q_2
q_2	coin	-	q_3
q_3	cof-but	cof	q_1
q_3	tea-but	tea	q_1

Inputs = {cof–but, tea–but, coin}
Outputs = {cof,tea}
States: $\{q_1,q_2,q_3\}$
Initial state = q_1
Transitions= {
$(q_1$, coin, –, $q_2)$,
$(q_2$, coin, –, $q_3)$,
$(q_3$, cof–but, cof, $q_1)$,
$(q_3$, tea–but, tea, $q_1)$
}

Sample run:

$$q_1 \xrightarrow{\text{coin/ –}} q_2 \xrightarrow{\text{coin/–}} q_3 \xrightarrow{\text{cof–but / cof}} q_1 \xrightarrow{\text{coin/ –}}$$

$$q_2 \xrightarrow{\text{coin/ –}} q_3 \xrightarrow{\text{cof–but / cof}} q_1$$

FSM as program 1

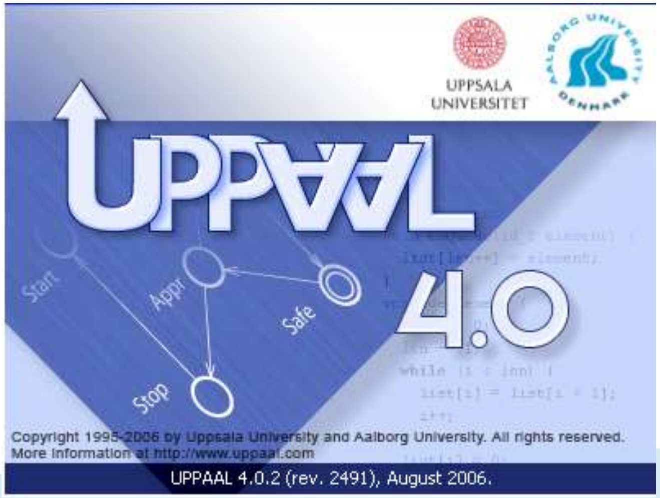
```
enum currentState {q1,q2,q3};
enum input {coin, cof_but,tea_but};
int nextStateTable[numStates][numInputs] = {
      q2,q1,q1,
      q3,q2,q2,
      q3,q1,q1 };

int outputTable[numStates][numInputs] = {
      0,0,0,
      0,0,0,
      coin,cof,tea};

While(Input=waitForInput()) {
  OUTPUT(outputTable[currentState,input])
  currentState=nextStateTable[currentState,input];

}
```

Adding Time

FSM

↓

Timed Automata



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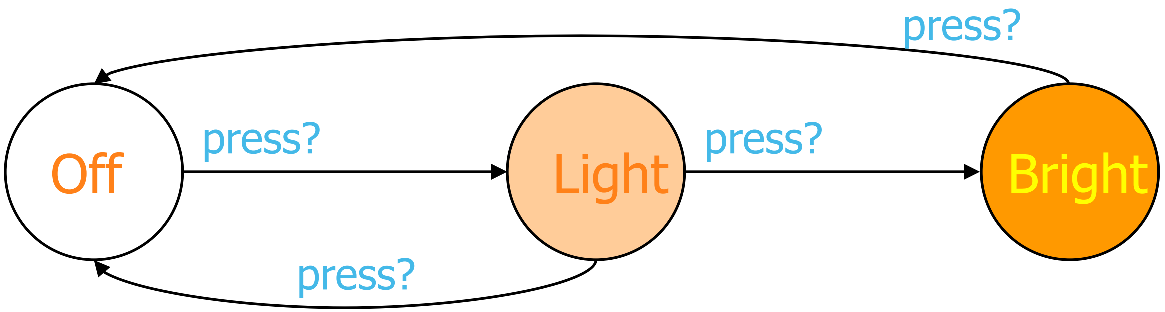
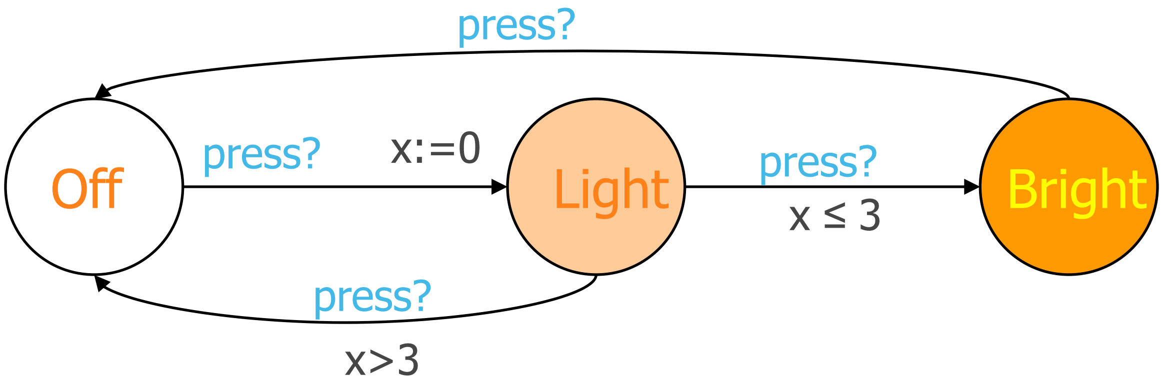
UPPAAL 4.0.2 (rev. 2491), August 2006.

Dumb Light Control



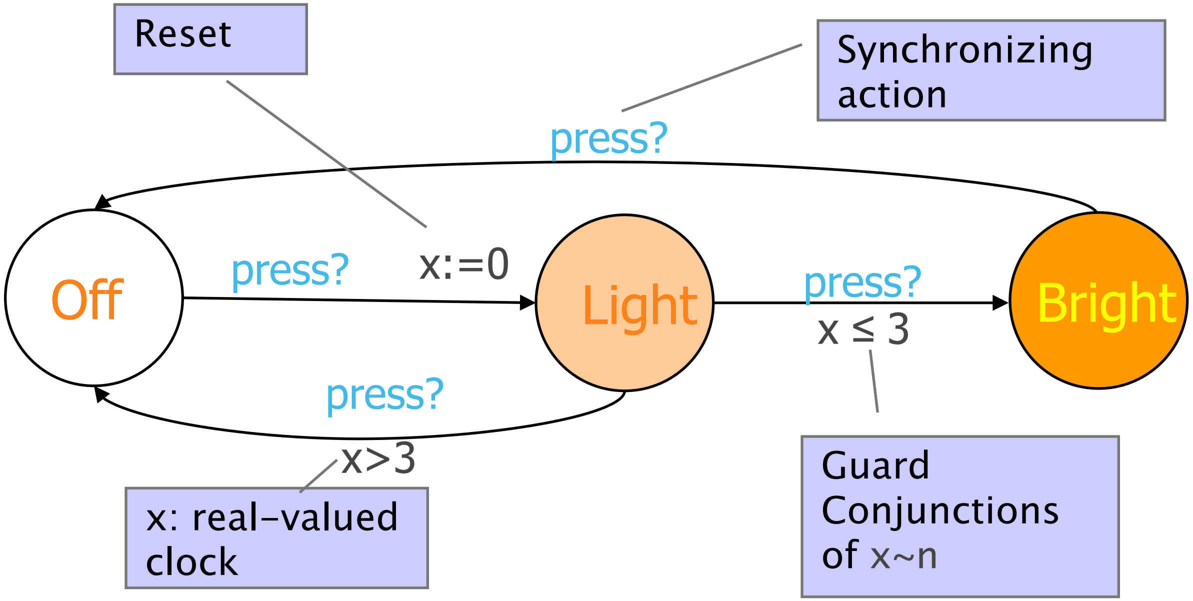
WANT: if press is issued twice quickly then the light will get brighter; otherwise the light is turned off.

Dumb Light Control



Solution: Add real-valued clock x to model the timing requirements: $|[\textit{quickly}]| = x \leq 3$

Timed Automata



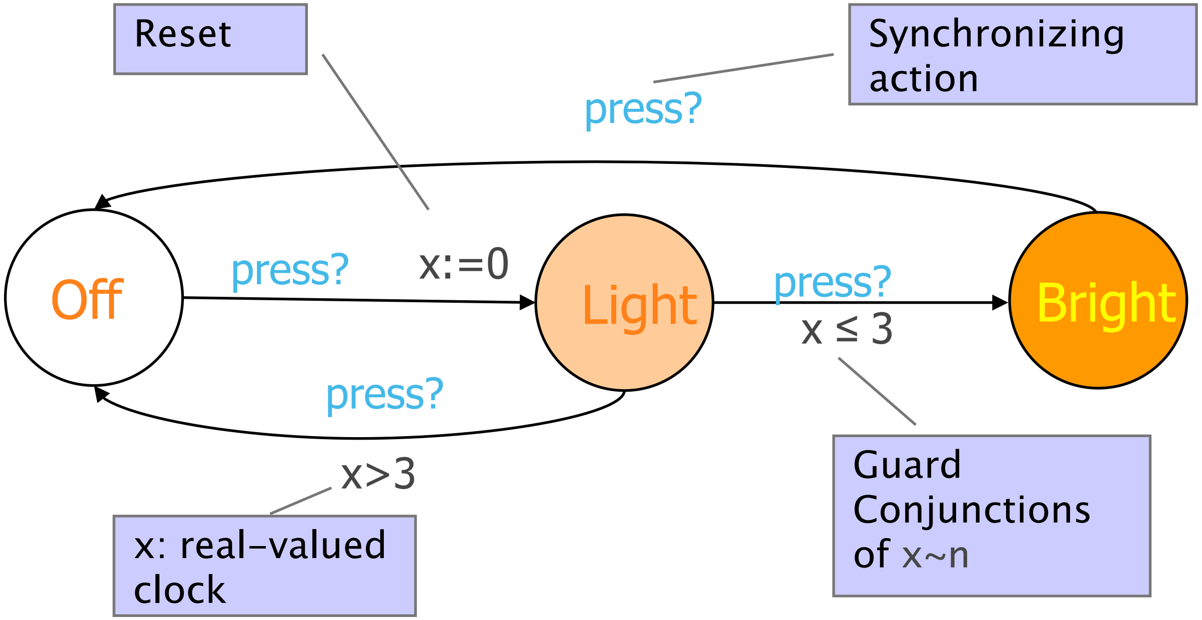
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

Timed Automata



States:

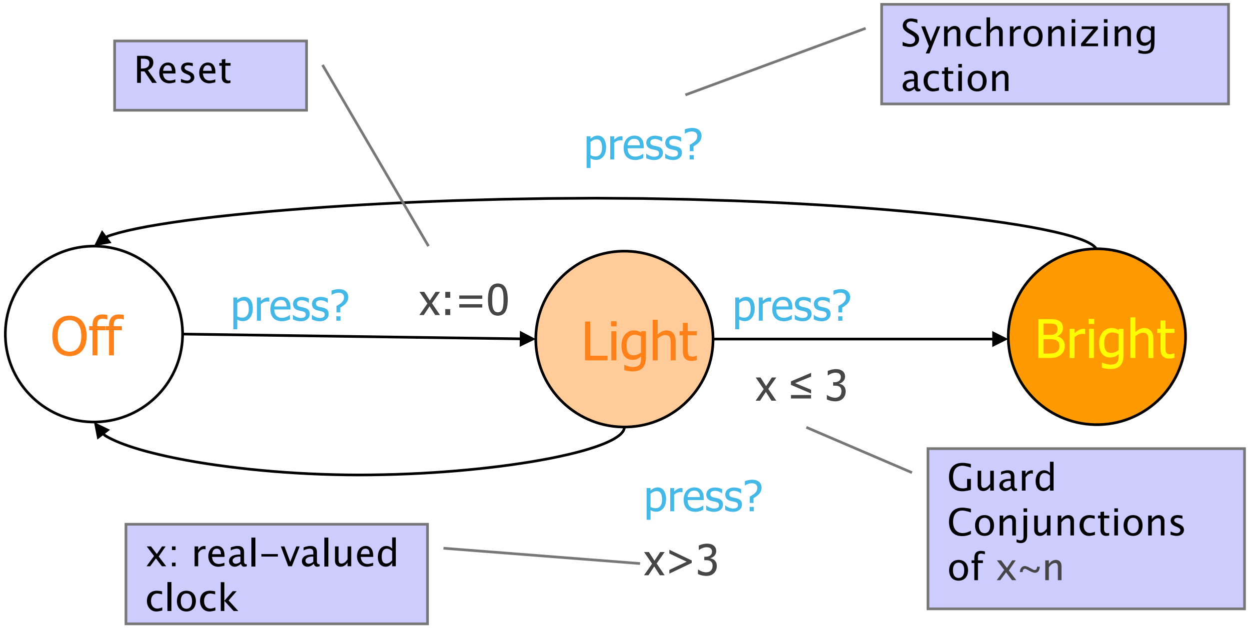
(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

Timed Automata



States:

(location , x=v) where v ∈ **R**

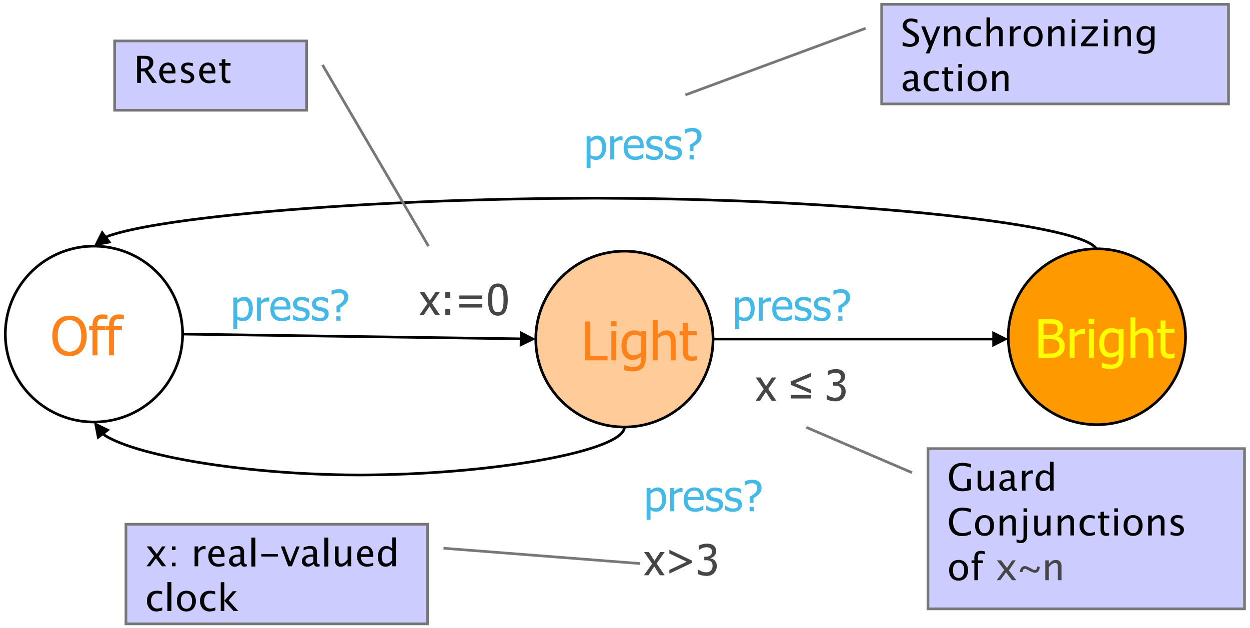
Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

press? → (Light , x=0)

Timed Automata



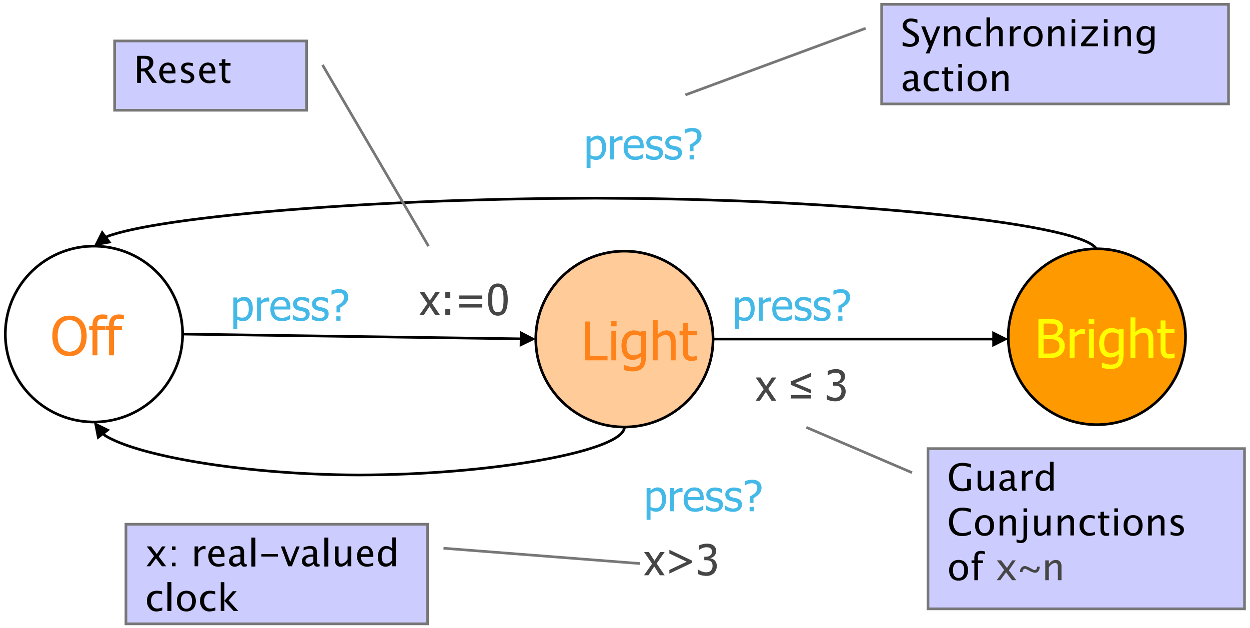
States:

(location , x=v) where v ∈ **R**

Transitions:

(Off , x=0)
delay 4.32 → (Off , x=4.32)
press? → (Light , x=0)
delay 2.51 → (Light , x=2.51)

Timed Automata



Reset

Synchronizing action

Guard
Conjunctions of x~n

x: real–valued clock

States:

(location , x=v) where v ∈ **R**

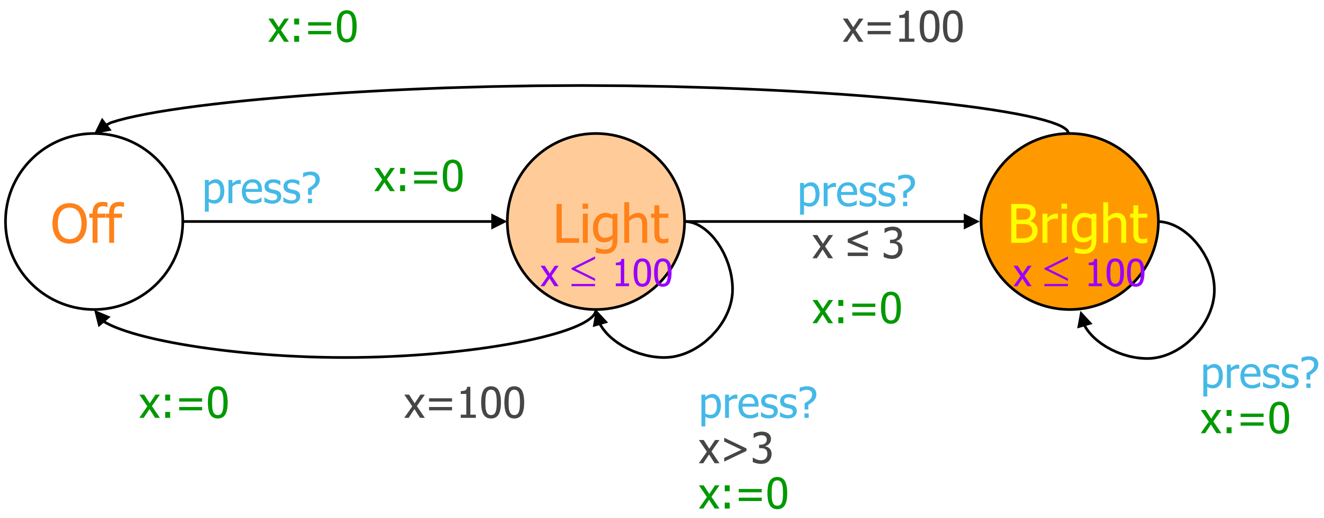
Transitions:

	(Off , x=0)
delay 4.32	→ (Off , x=4.32)
press?	→ (Light , x=0)
delay 2.51	→ (Light , x=2.51)
press?	→ (Bright , x=2.51)

Intelligent Light Control

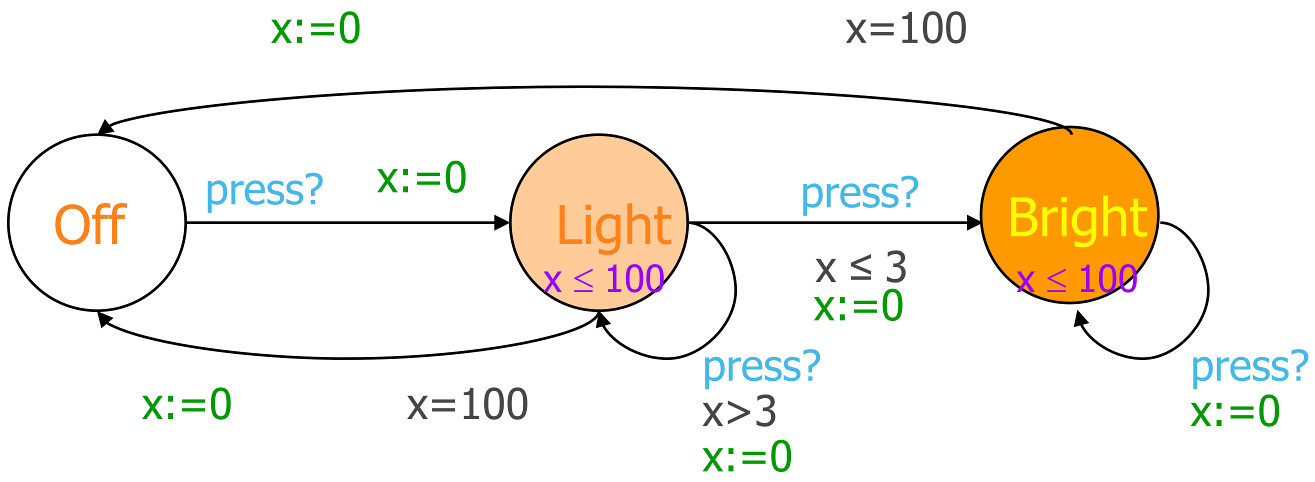
Using Invariants

Requirement: automatically switch light off after 100 time units



Intelligent Light Control

Using Invariants



Transitions:

(Off , x=0)

delay 4.32 → (Off , x=4.32)

press? → (Light , x=0)

delay 4.51 → (Light , x=4.51)

press? → (Light , x=0)

delay 100 → (Light , x=100)

τ → (Off , x=0)

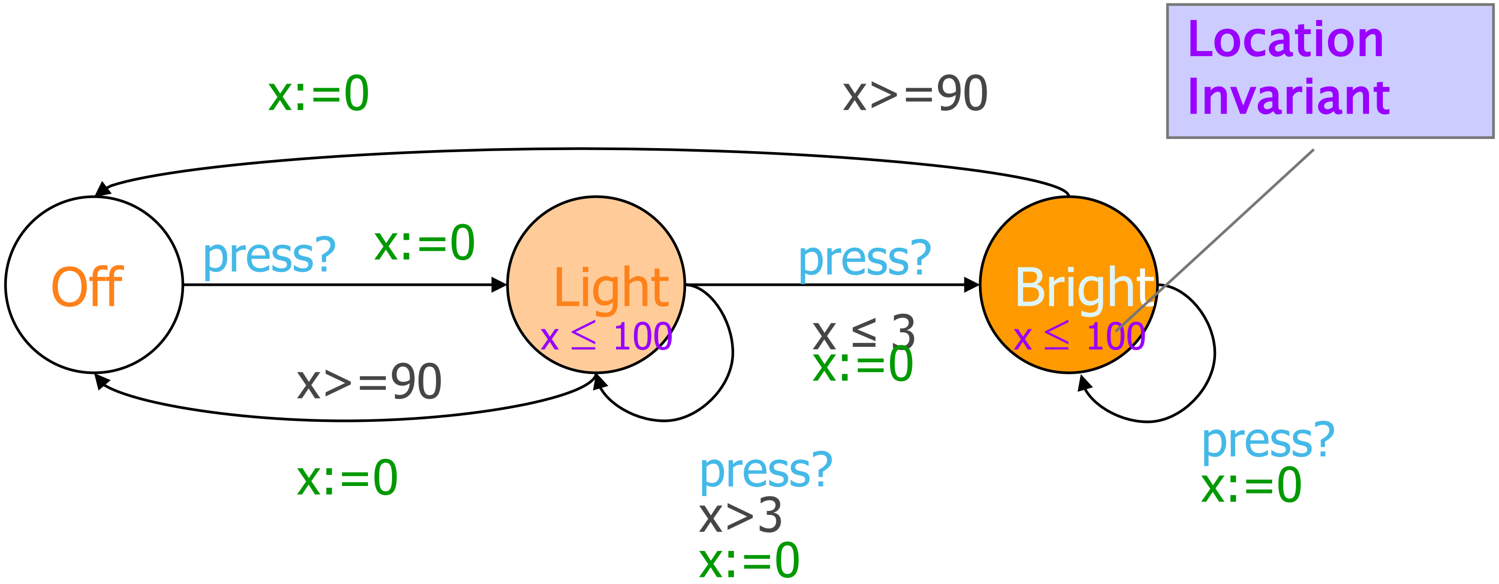
Note:

(Light , x=0) delay 103 → X

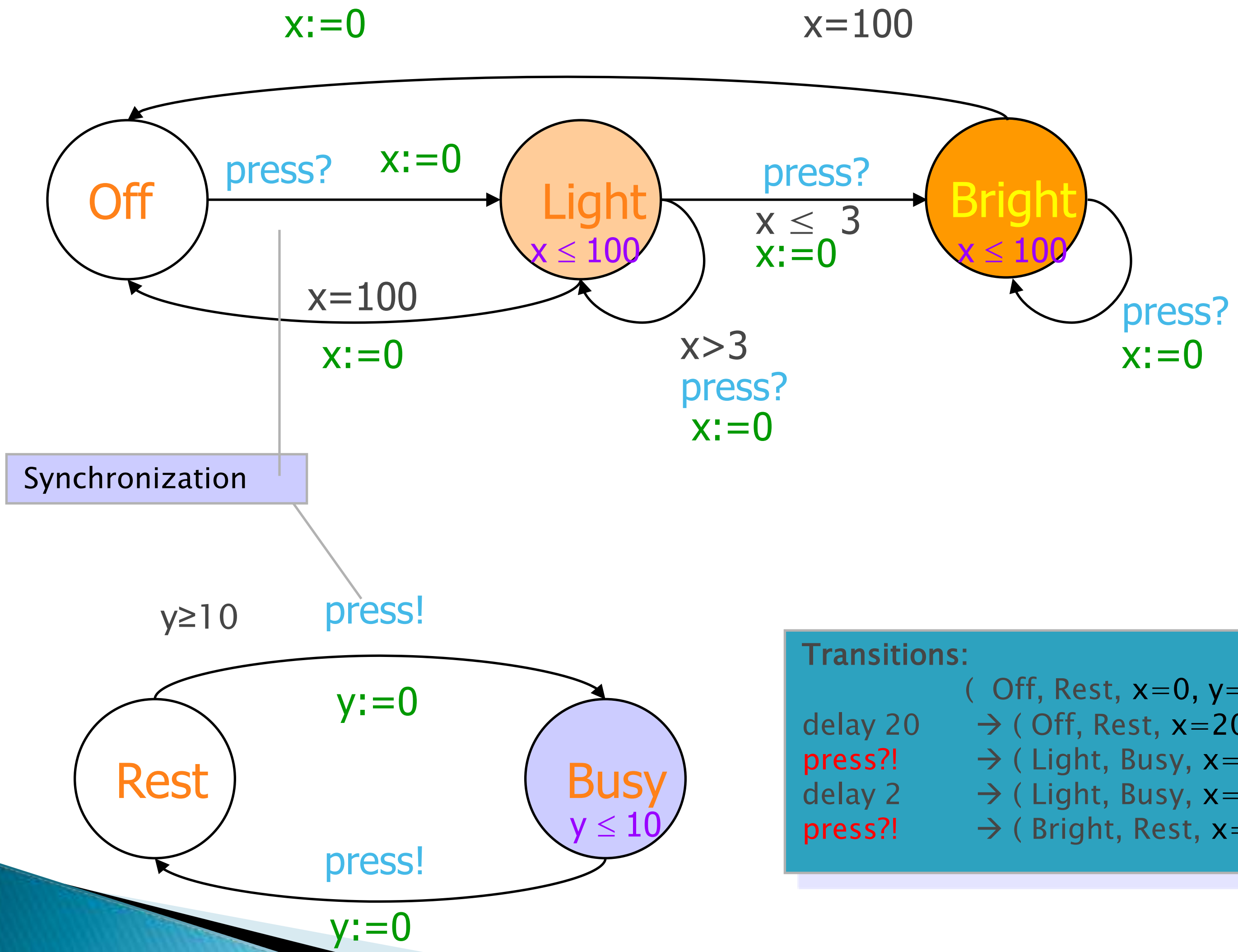
Invariants ensures progress

Intelligent Light Control

Requirements including uncertainty:
Automatically switch light off after ***between*** 90–100 time units



Light Controller || User

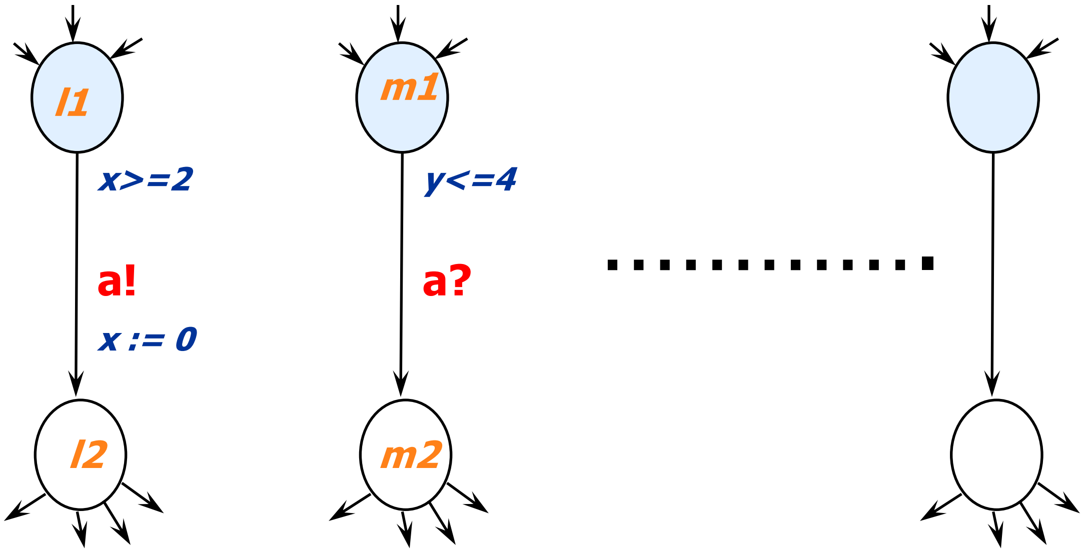


Transitions:

	(Off, Rest, x=0, y=0)
delay 20	→ (Off, Rest, x=20, y=20)
press?!	→ (Light, Busy, x=0, y=0)
delay 2	→ (Light, Busy, x=2, y=2)
press?!	→ (Bright, Rest, x=0, y=0)

Networks of Timed Automata

(a'la CCS)



Two-way synchronization on *complementary* actions.

Closed Systems!

Example transitions

$(l1, m1, \ldots, x=2, y=3.5, \ldots) \xrightarrow{tau} (l2, m2, \ldots, x=0, y=3.5, \ldots)$

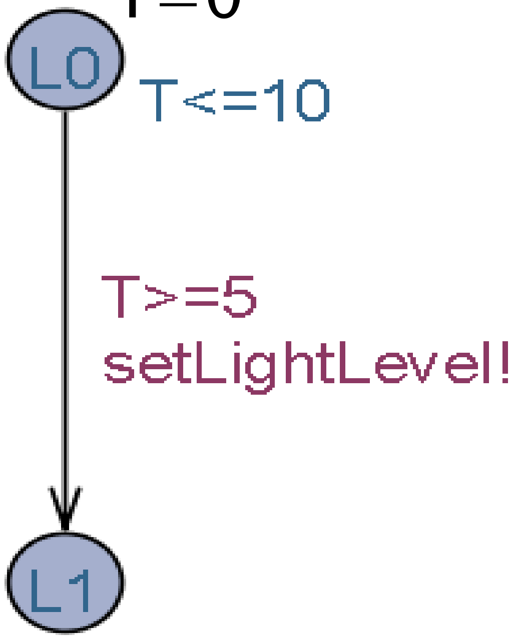
$(l1, m1, \ldots, x=2, y=3.5, \ldots) \cancel{\xrightarrow{0.2}} (l1, m1, \ldots, x=2.2, y=3.7, \ldots)$

If **a** URGENT CHANNEL

Timing Uncertainty

- Unpredictable or variable
  - response time,
  - computation time
  - transmission time etc:

Initially
T=0

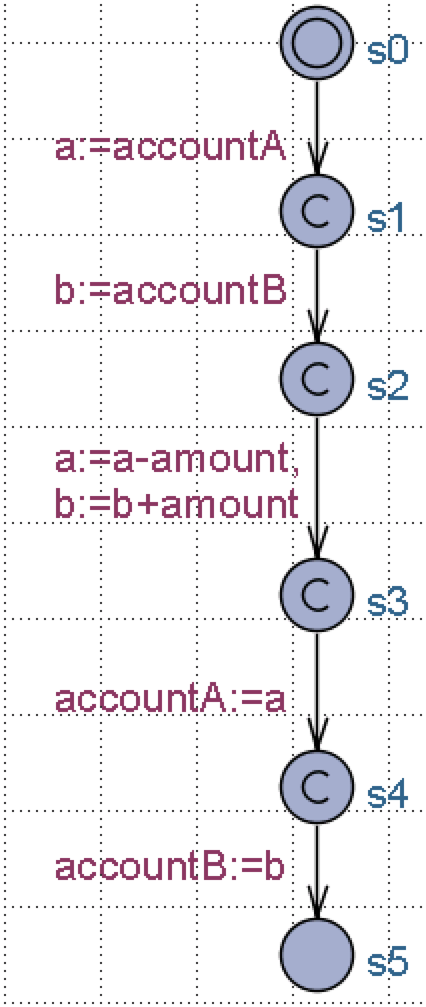


LightLevel must be adjusted
between 5 and 10

Comitted Locations

- Locations marked C
  - *No delay* in committed location.
  - No interleaving with parallel transitions
- Handy to model atomic sequences
- The use of committed locations reduces the number of states in a model, and allows for more space and time efficient analysis.

- S0 to s5 executed atomically



Urgent Channels and Locations

- Locations marked U
  - *No delay* like in committed location.
  - But Interleaving permitted
- Channels declared "**`urgent chan`**"
  - Time doesn't elapse when a synchronization is possible on a pair of urgent channels
  - Interleaving allowed

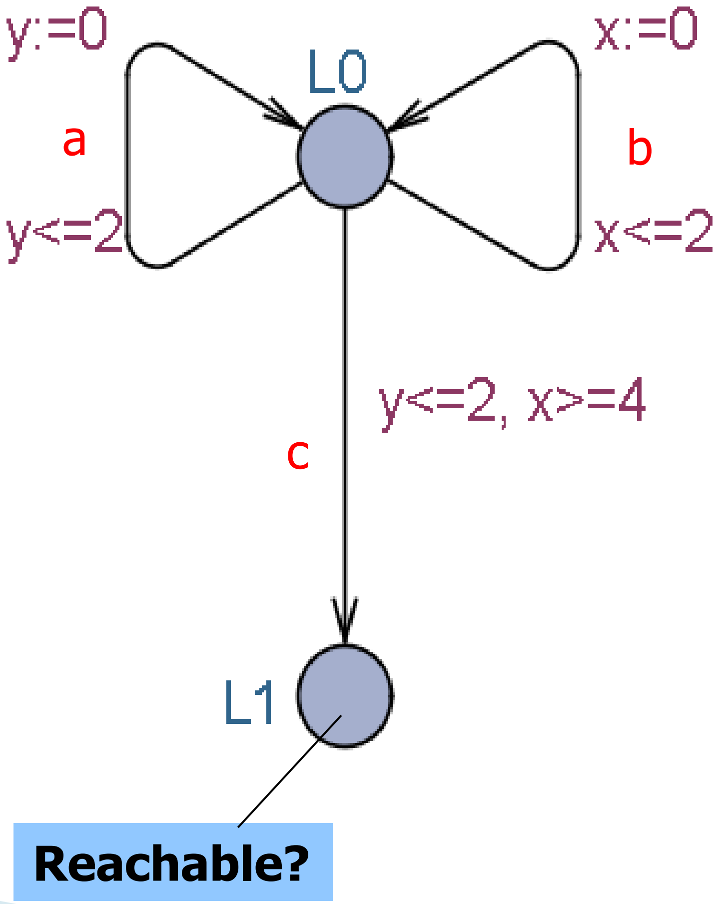
Broad-casts

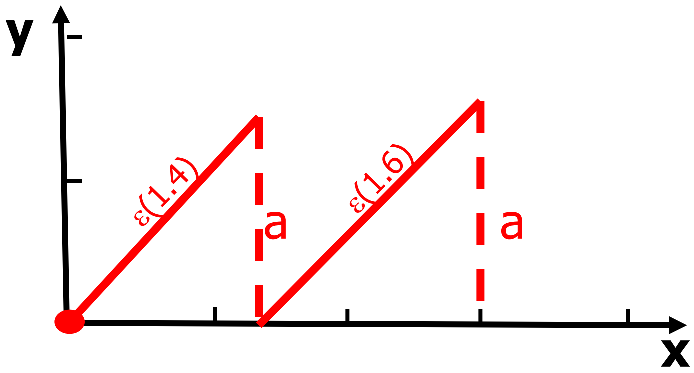
- chan coin, cof, cofBut;
- broadcast chan join;
    - sending: output join!
    - every automaton that listens to join moves on
    - ie. every automaton with enabled "join?" transition moves in one step
    - may be zero! Listeners, sender can progress anyway

Other Uppaal features

- Bounded domains
  - `int [1..4] a;`
- C-like data-structures and user defined functions in declaration section
  - structs, arrays, and typedef
- non-deterministic assignment:
  - `select a:T`
- `forall`, `exists` in expressions
- Scalar sets (for giving unique ID's)
- Process and channel **priorities**
- Value passing (emulation)

Timed traces





(L0,x=0,y=0)

$\rightarrow_{\varepsilon(1.4)}$

(L0,x=1.4,y=1.4)

$\rightarrow_{a}$

(L0,x=1.4,y=0)

$\rightarrow_{\varepsilon(1.6)}$

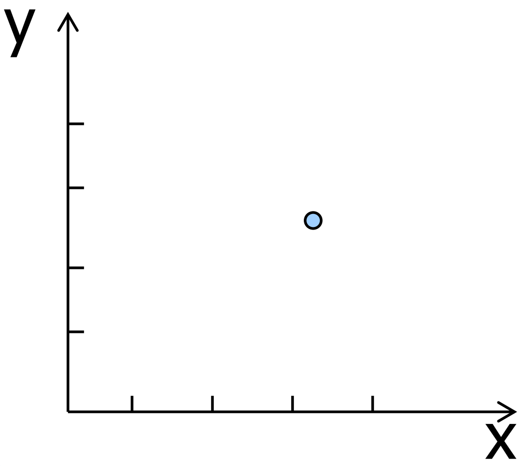
(L0,x=3.0,y=1.6)

$\rightarrow_{a}$

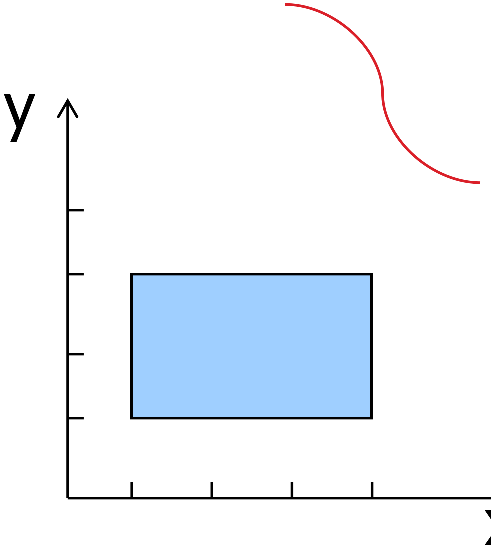
(L0,x=3.0,y=0)

From explicit clock values to zones
(*from infinite to finite*)

Explicit state
(n, x=3.2, y=2.5)

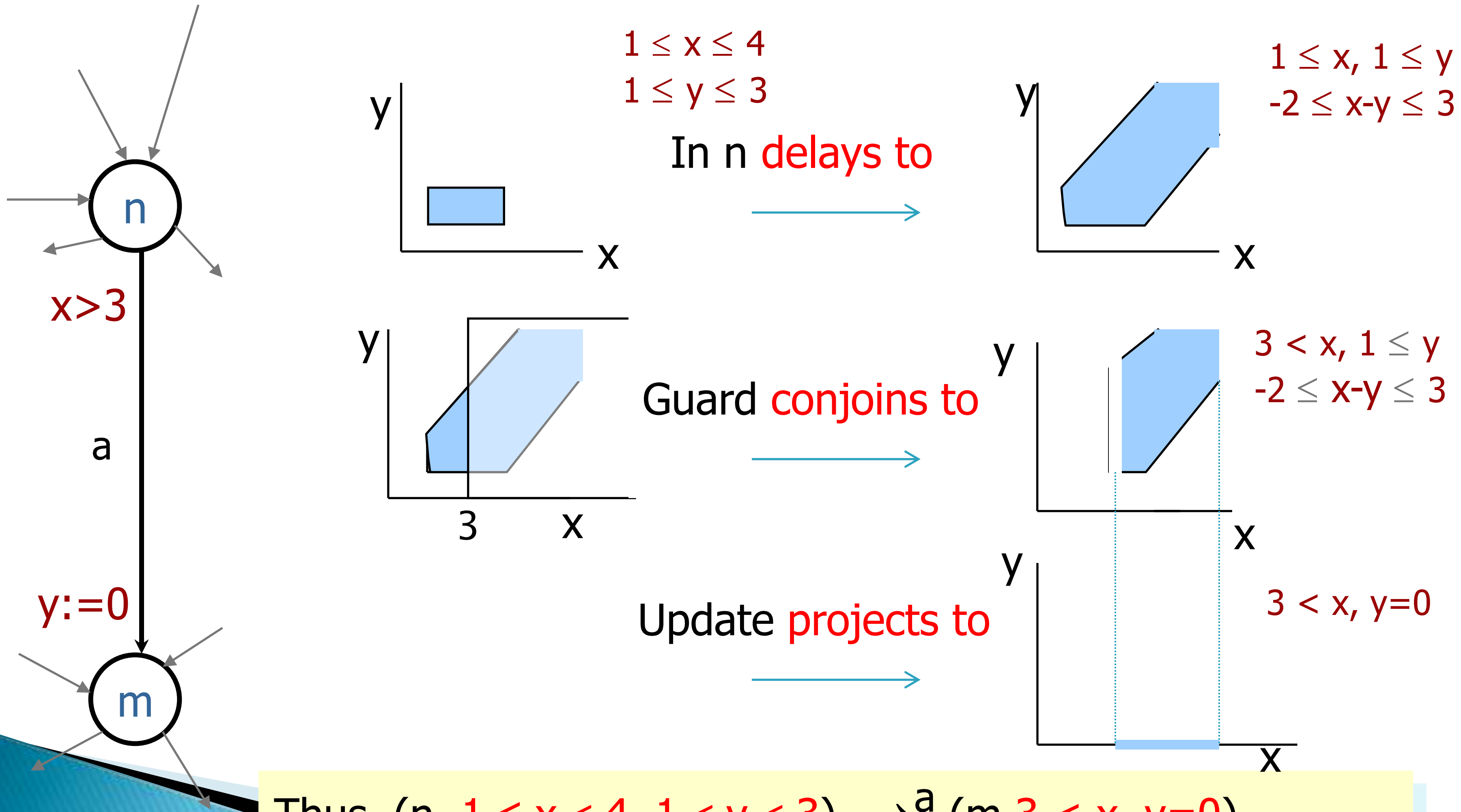


Symbolic state (set)
(n, $1 \leq x \leq 4$, $1 \leq y \leq 3$)



Zone:
conjunction of
clock constraints
of form:
x-y<=const1,
x<=const2,
x>=const3

Symbolic Transitions



$1 \le x \le 4$
$1 \le y \le 3$

In n delays to

$1 \le x, 1 \le y$
$-2 \le x-y \le 3$

Guard conjoins to

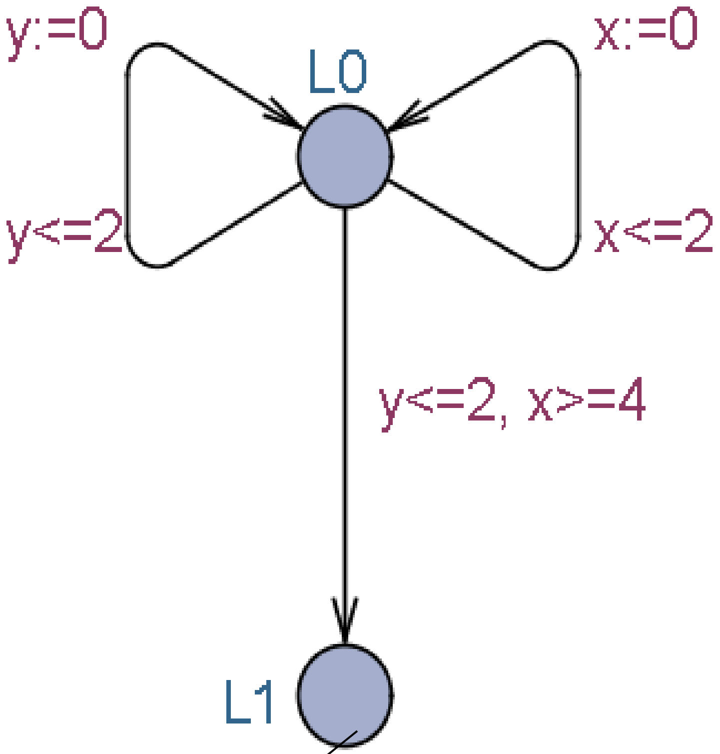
$3 < x, 1 \le y$
$-2 \le x-y \le 3$

Update projects to

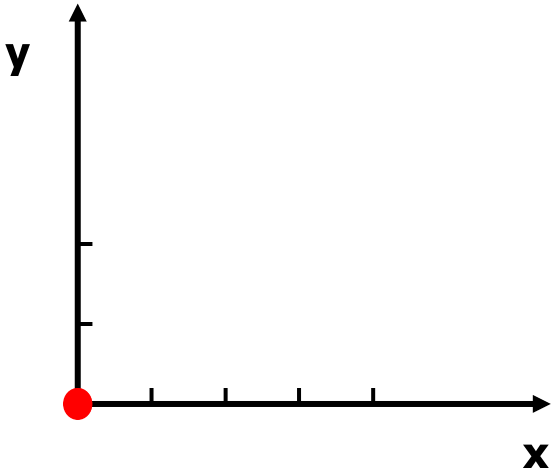
$3 < x, y=0$

Thus $(n, 1 \le x \le 4, 1 \le y \le 3) \rightarrow^{a} (m, 3 < x, y=0)$

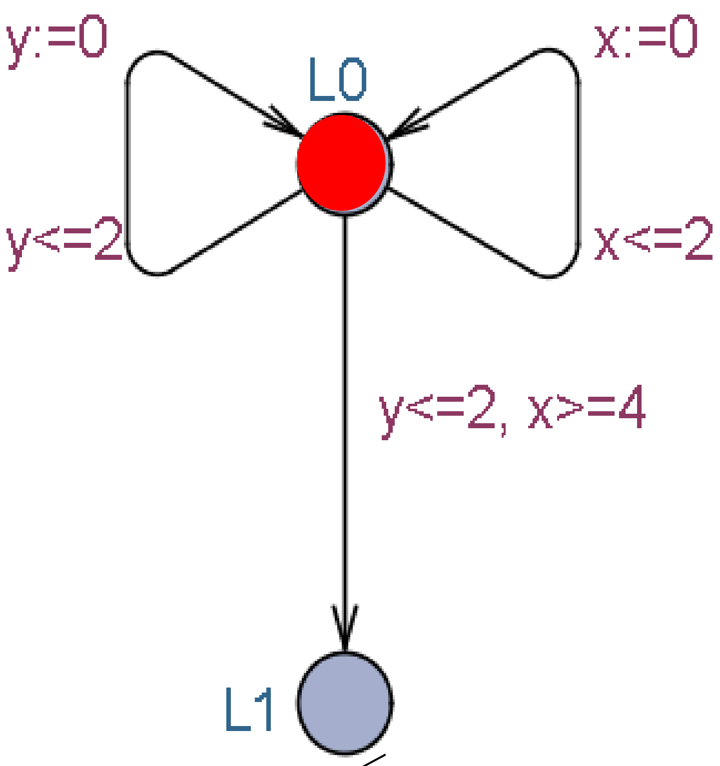
Symbolic Exploration



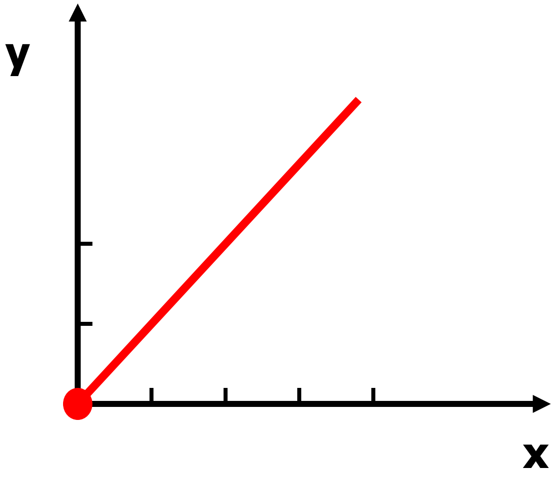
Reachable?



Symbolic Exploration

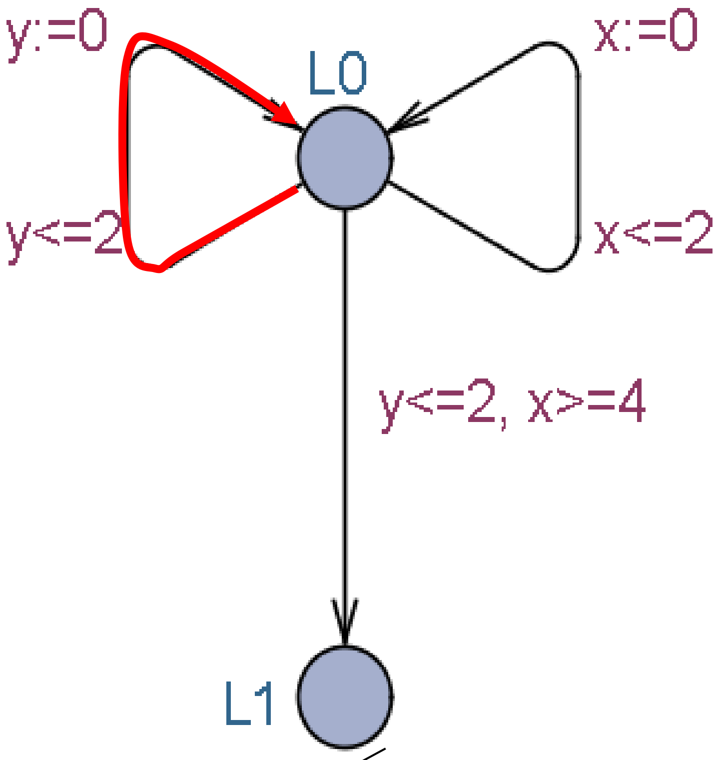


Reachable?

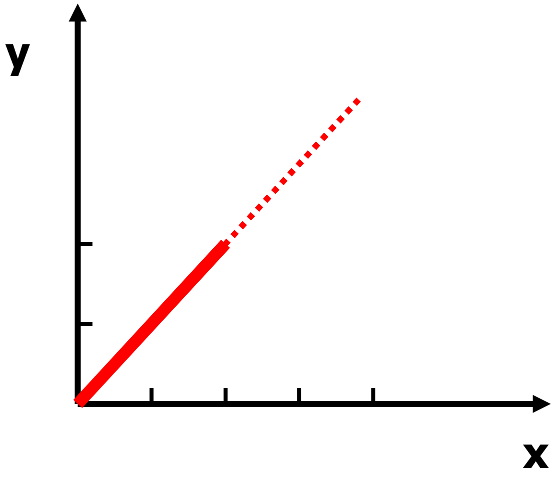


Delay

Symbolic Exploration

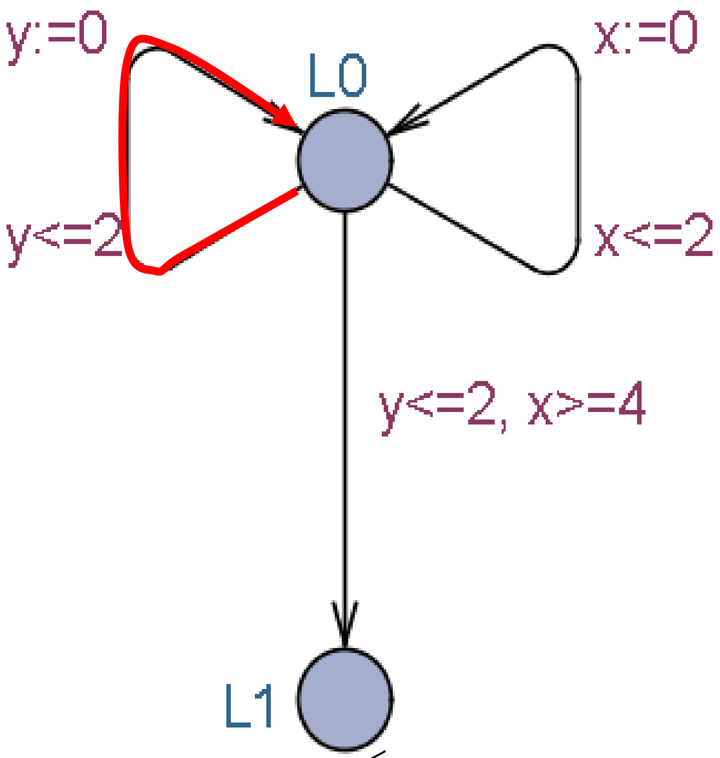


Reachable?

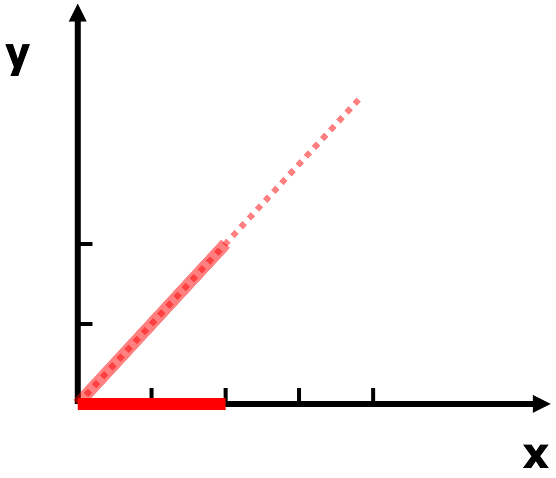


Left

Symbolic Exploration

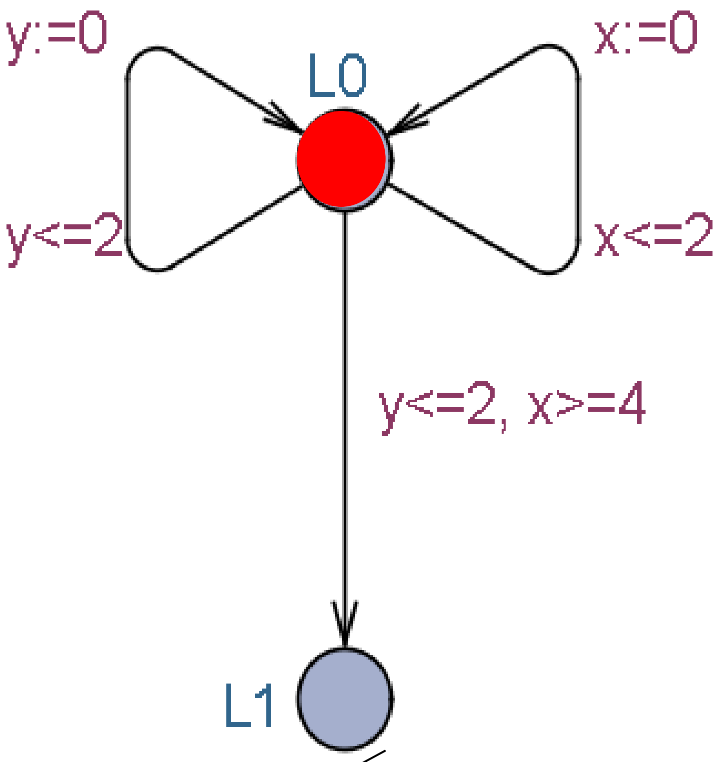


Reachable?

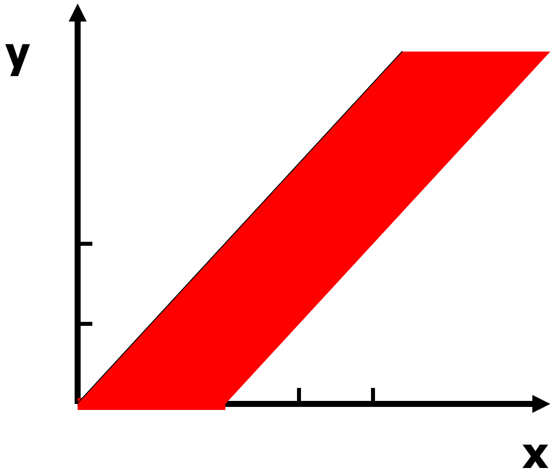


Left

Symbolic Exploration

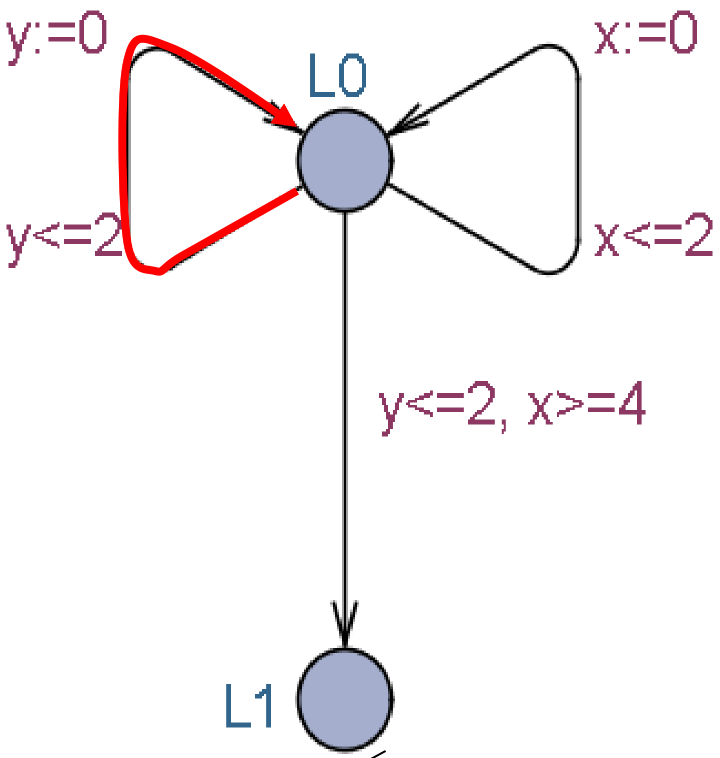


Reachable?



Delay

Symbolic Exploration



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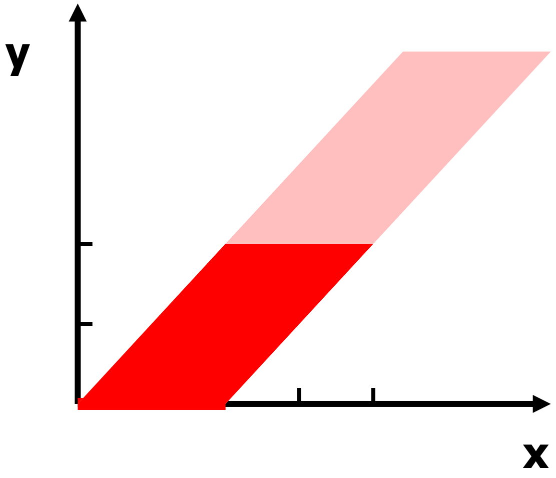
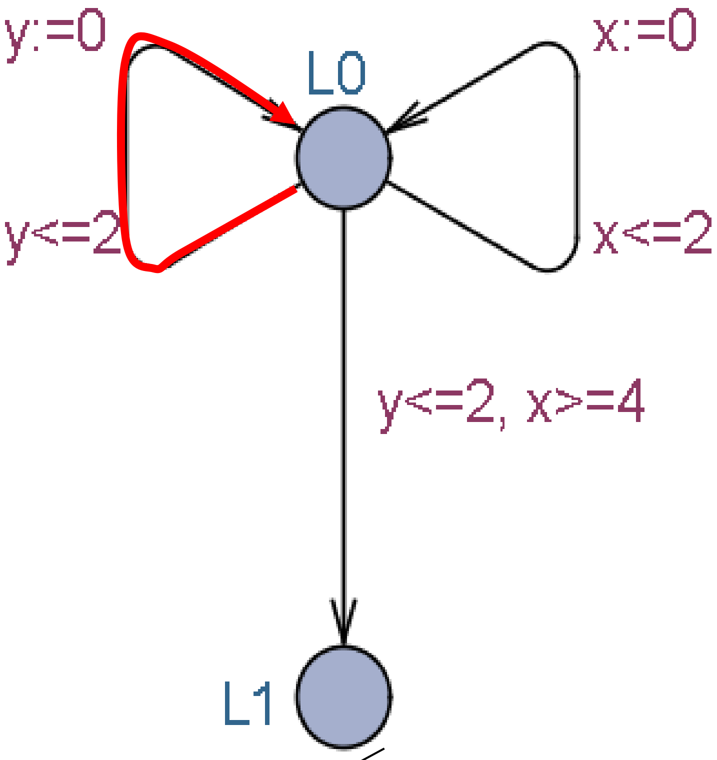


Left

Symbolic Exploration



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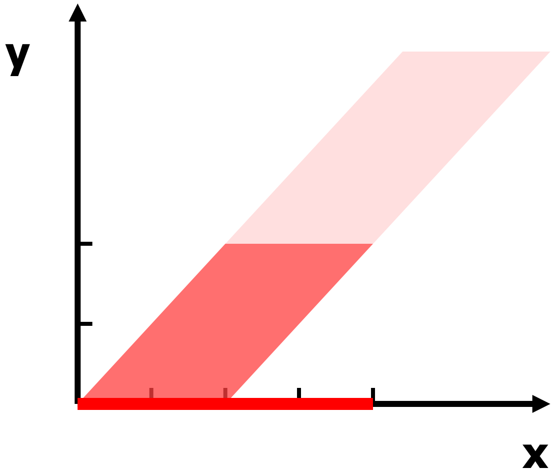
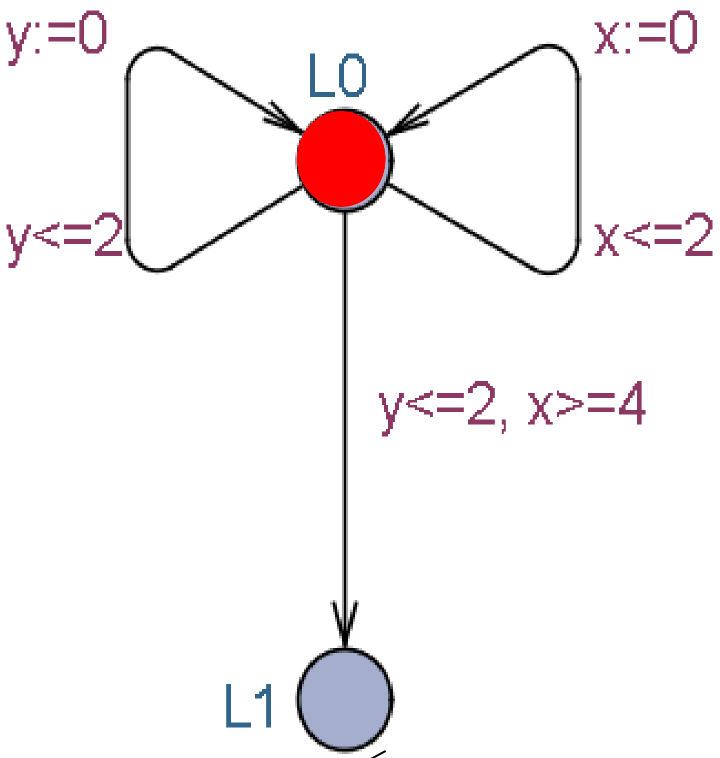


Left

Symbolic Exploration



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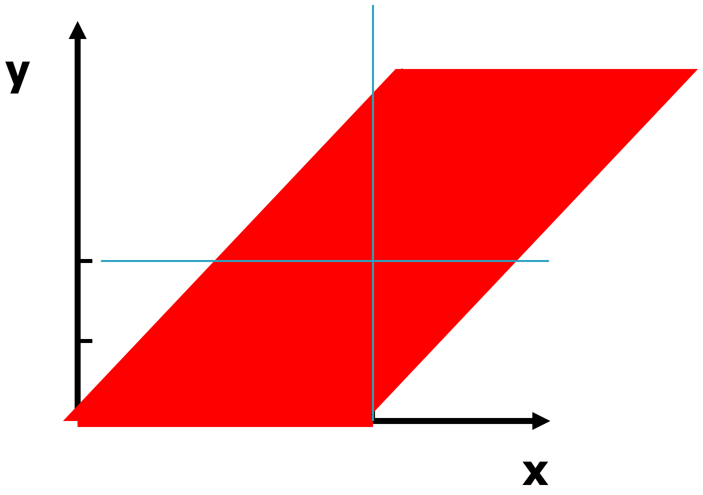
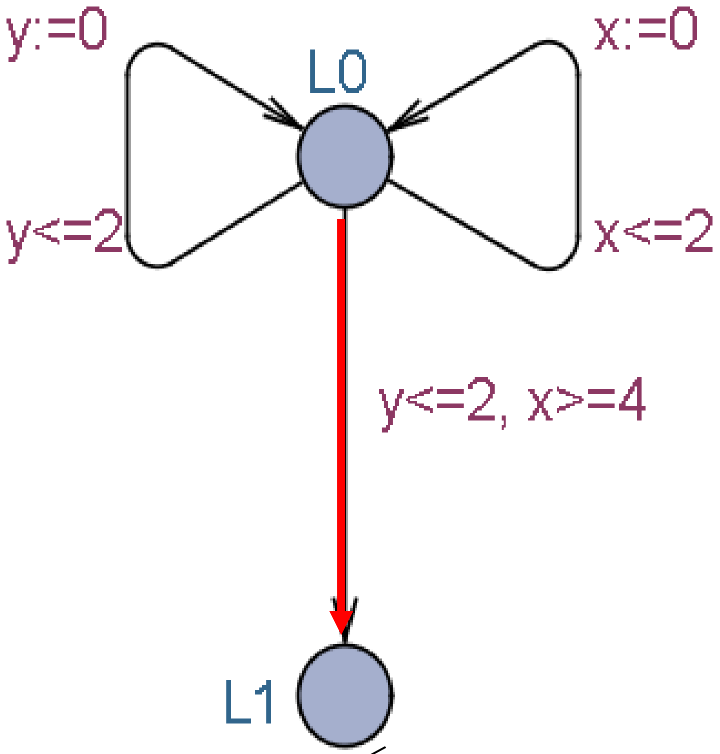
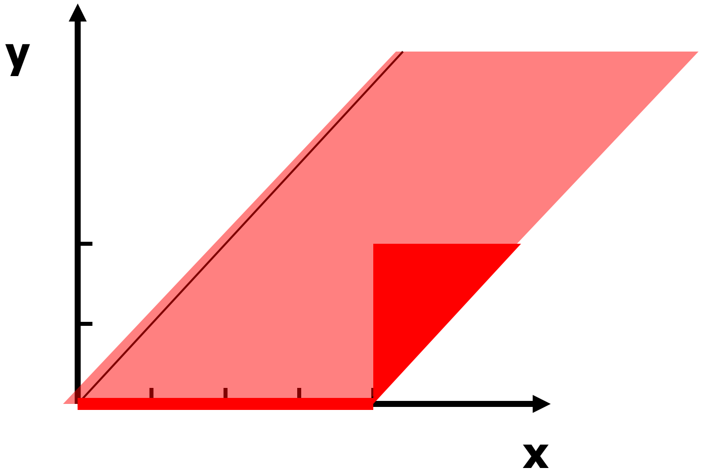


Delay

Symbolic Exploration



Reachable?

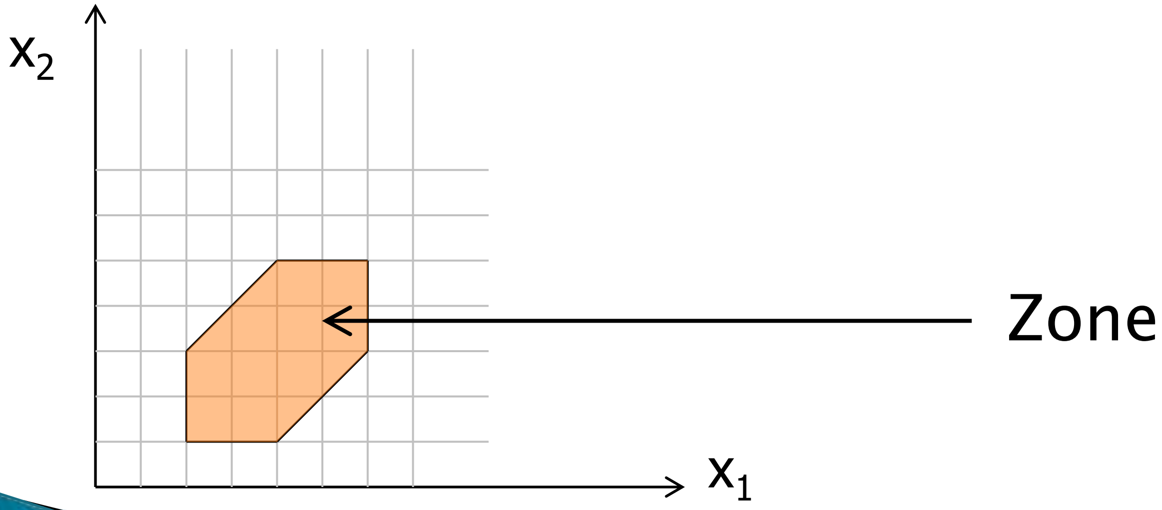


Down

Difference Bound Matrices

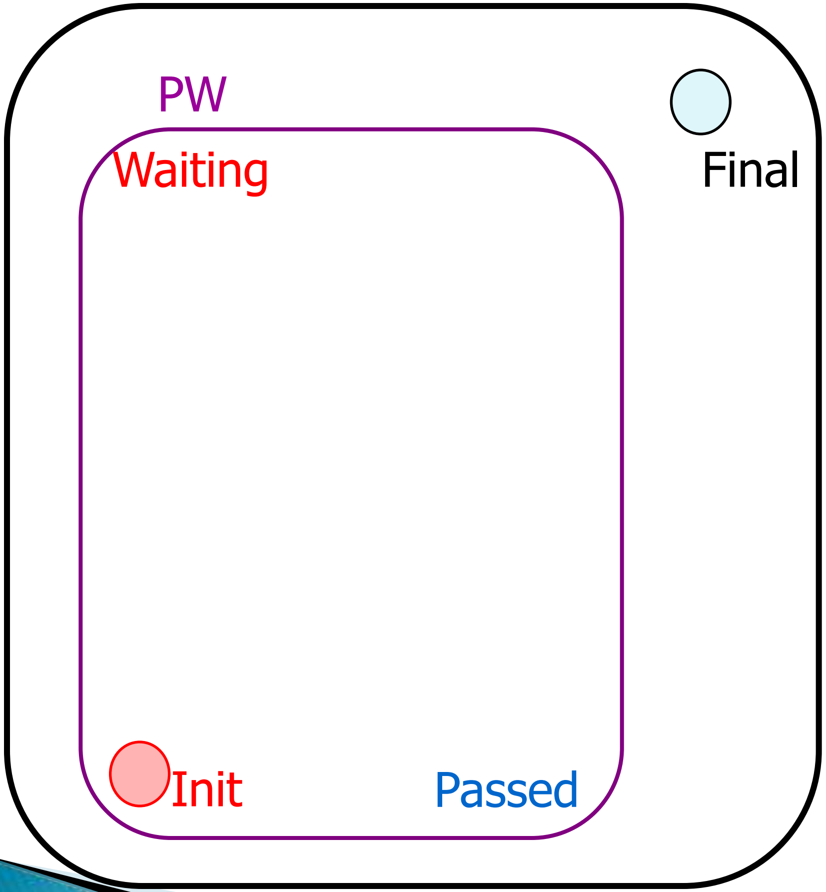
$x_0-x_0<=0$	$x_0-x_1<=-2$	$x_0-x_2<=-1$
$x_1-x_0<=6$	$x_1-x_1<=0$	$x_1-x_2<=3$
$x_2-x_0<=5$	$x_2-x_1<=1$	$x_2-x_2<=0$

$x_i-x_j<=c_{ij}$



Forward Reachability Algorithm

Init -> Final ?



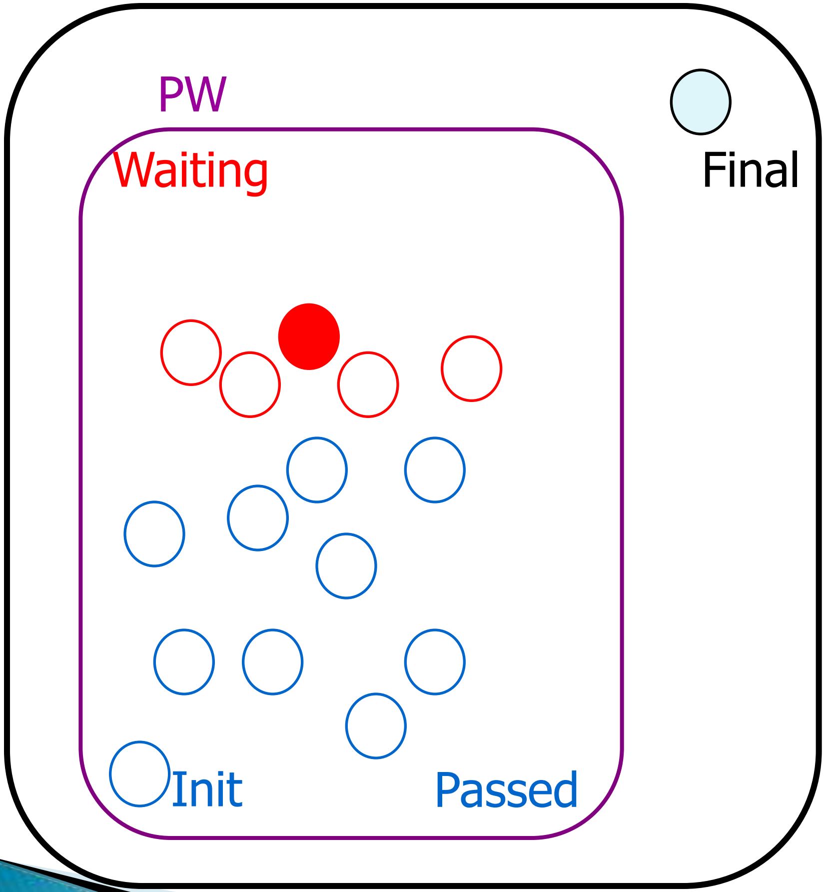
```
INITIAL  Passed := Ø;
         Waiting := {($n_0$,$Z_0$)}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all (n,Z)→(n′,Z′):
      if for some (n′,Z″) Z′⊆ Z″ continue
      else add (n′,Z′) to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



```
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            Waiting := {($n_0$,$Z_0$)}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all (n,Z)→(n′,Z′):
      if for some (n′,Z″) $Z' \subseteq Z''$ continue
      else add (n′,Z′) to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```
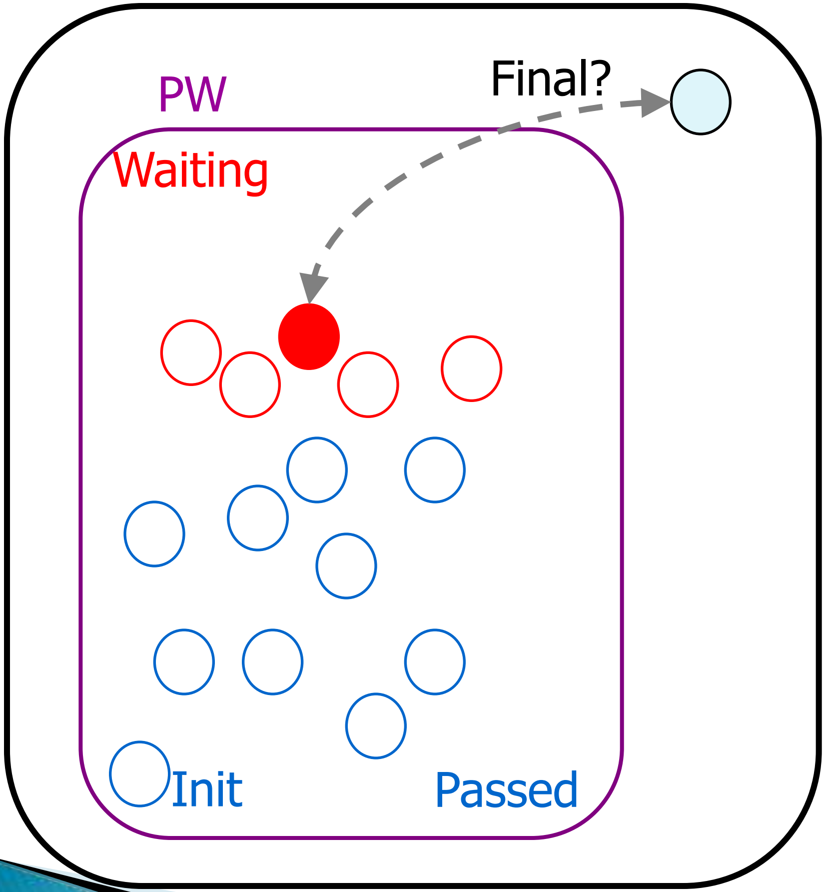

Forward Reachability Algorithm

Init -> Final ?



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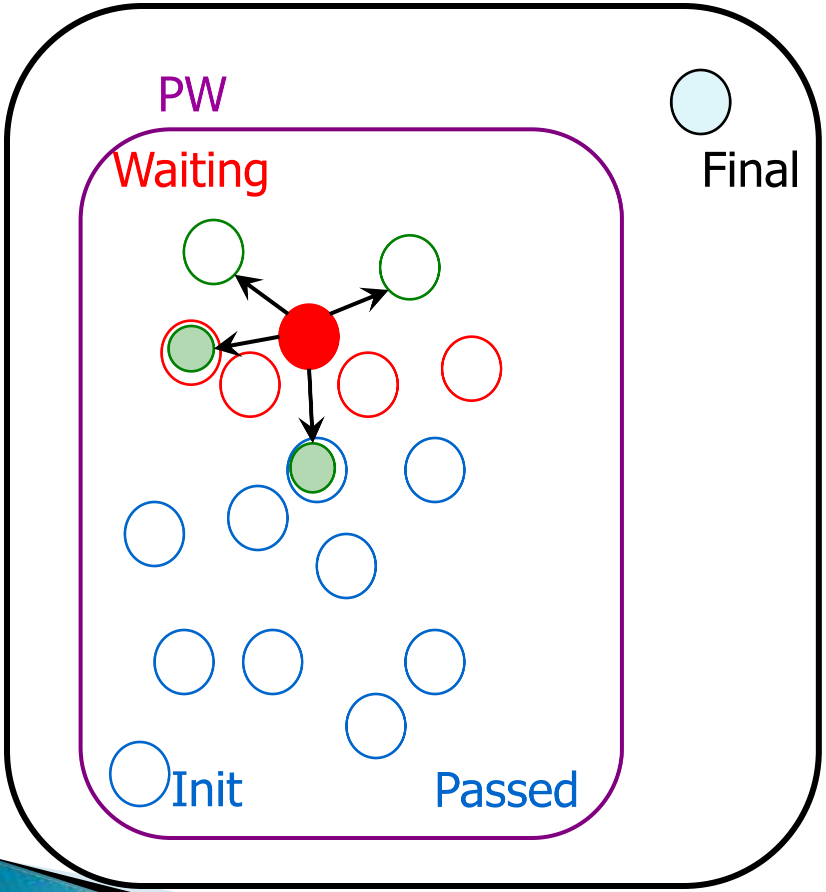
              Waiting := {($n_0$,$Z_0$)}

REPEAT
    pick (n,Z) in Waiting
    if (n,Z) = Final return true
    for all (n,Z)→(n',Z'):
        if for some (n',Z'') Z' ⊆ Z'' continue
        else add (n',Z') to Waiting
        move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



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REPEAT
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   if (n,Z) = Final return true
   for all (n,Z)→(n',Z'):
      if for some (n',Z'') $Z' \subseteq Z''$ continue
      else add (n',Z') to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```
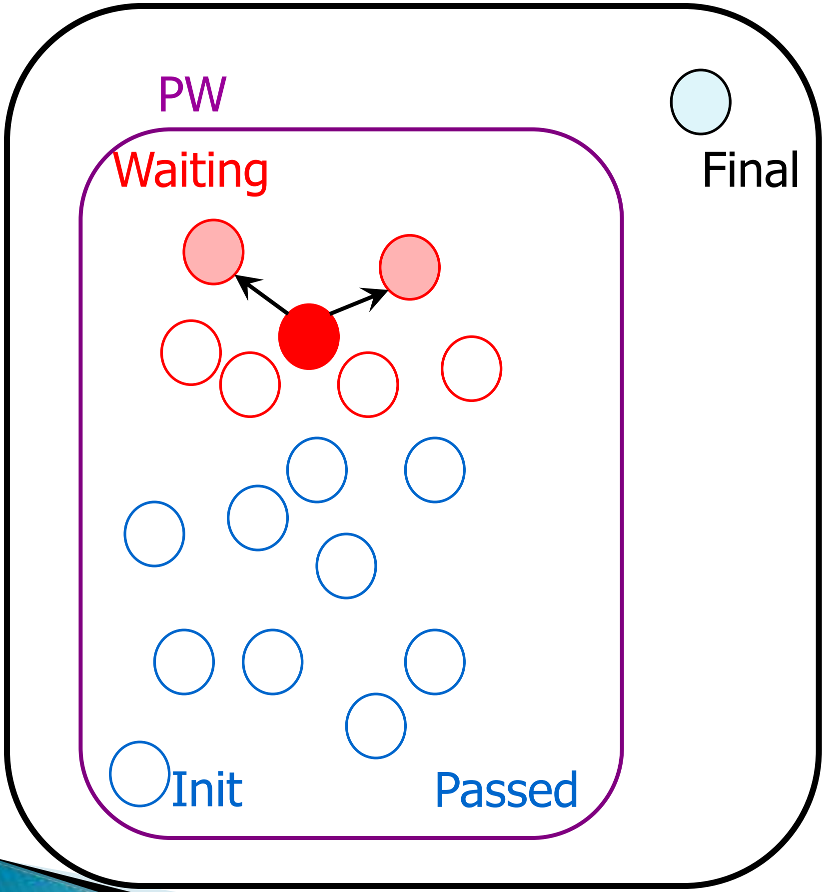

Forward Reachability Algorithm

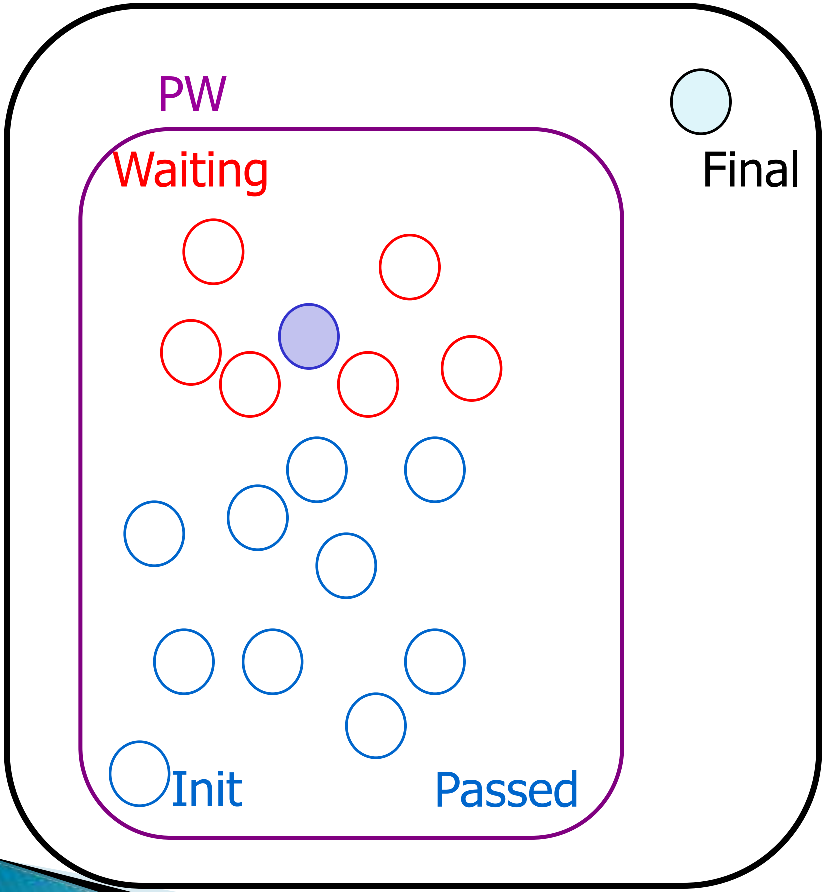
Init -> Final ?



```
INITIAL  Passed := Ø;
         Waiting := {$(n_0,Z_0)$}

REPEAT
   pick (n,Z) in Waiting
   if (n,Z) = Final return true
   for all $(n,Z) \rightarrow (n',Z')$:
      if for some (n',Z'') $Z' \subseteq Z''$ continue
      else add (n',Z') to Waiting
      move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

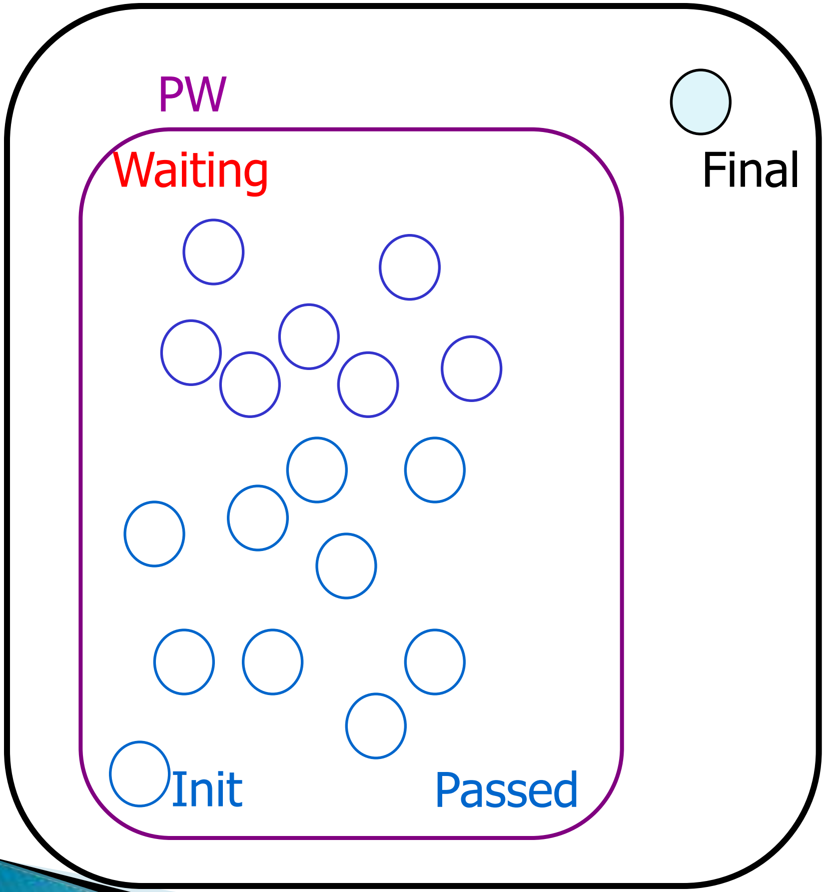
Init -> Final ?



```
INITIAL  Passed := Ø;
         Waiting := {($n_0$,$Z_0$)}

REPEAT
  pick (n,Z) in Waiting
  if (n,Z) = Final return true
  for all (n,Z)$\rightarrow$(n',Z'):
    if for some (n',Z'') $Z' \subseteq Z''$ continue
    else add (n',Z') to Waiting
    move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```

Forward Reachability Algorithm

Init -> Final ?



```
INITIAL  Passed := Ø;
           Waiting := {$(n_0,Z_0)$}

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  for all $(n,Z)\rightarrow(n',Z')$:
    if for some $(n',Z'')$ $Z' \subseteq Z''$ continue
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    move (n,Z) to Passed

UNTIL  Waiting = Ø
return false
```
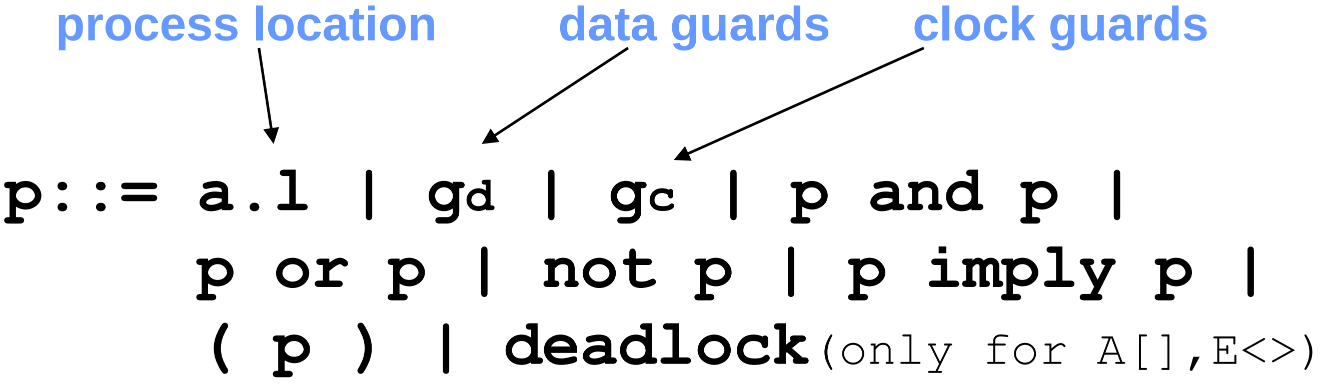

Specification (Query) Language

UPPAAL Property Specification Language

- **`A[] p`** *always*
- **`A<> p`** *inevitable*
- **`E<> p`** *Possible*
- **`E[] p`** *potentially always*
- **`P --> q`** *leads-to*



process location

data guards

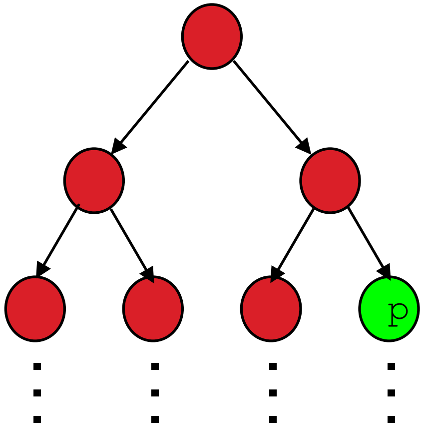
clock guards

p::= a.l | g_d | g_c | p and p |
p or p | not p | p imply p |
(p) | deadlock(only for A[],E<>)

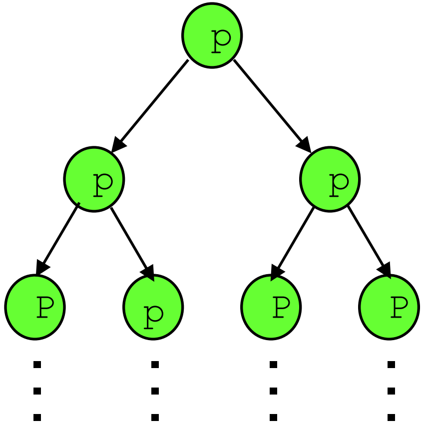
`A[] (mc1.finished and mc2.finished) imply (accountA+accountB==200)`

Uppaal “Computation Tree Logic”

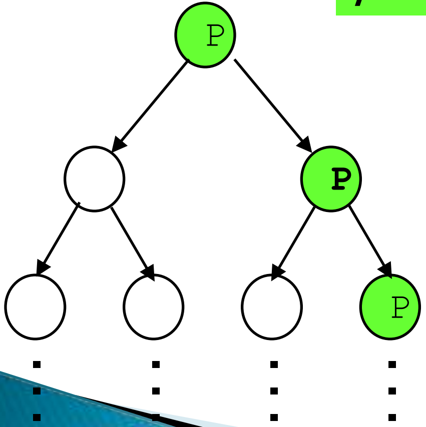
`E<> p` *Possible*



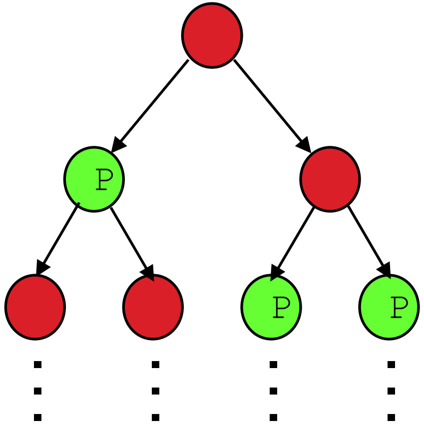
`A[] p` *always*



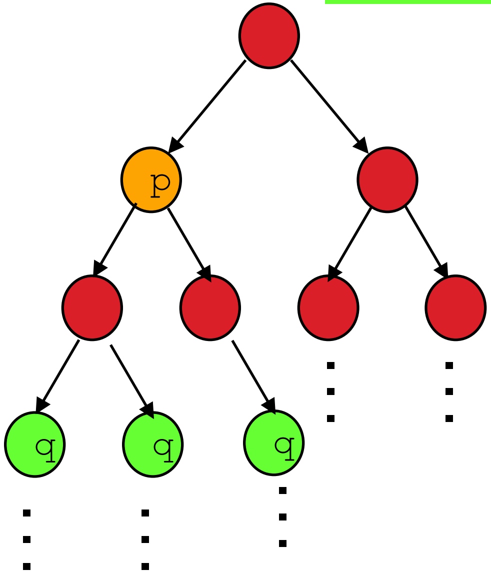
`E[] p` *potentially always*



`A<> p` *inevitable*



`p --> q` *leads-to*



Logical Specifications

- Validation Properties
    - Possibly: E<> *P*
- Safety Properties
    - Invariant: A[] *P*
    - Pos. Inv.: E[] *P*
- Liveness Properties
    - Eventually: A<> *P*
    - Leadsto: P --> *Q*
- Bounded Liveness
    - Leads to within: *P* $-\!\!->_{\leq t}$ *Q*

The expressions *P* and *Q* must be type safe, side effect free, and evaluate to a boolean.

Only references to integer variables, constants, clocks, and locations are allowed (and arrays of these).

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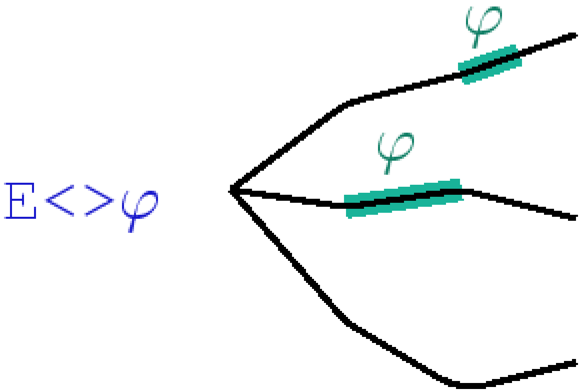

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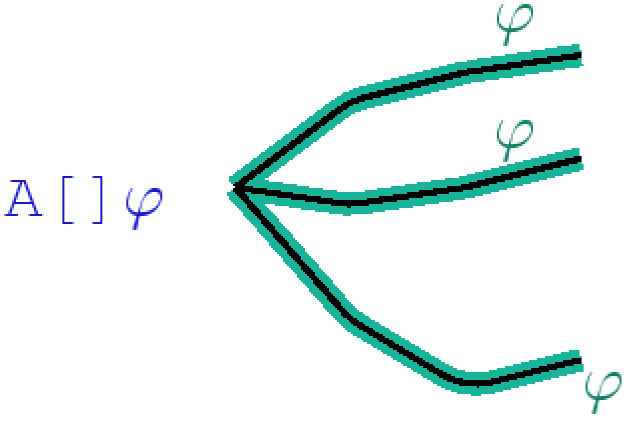

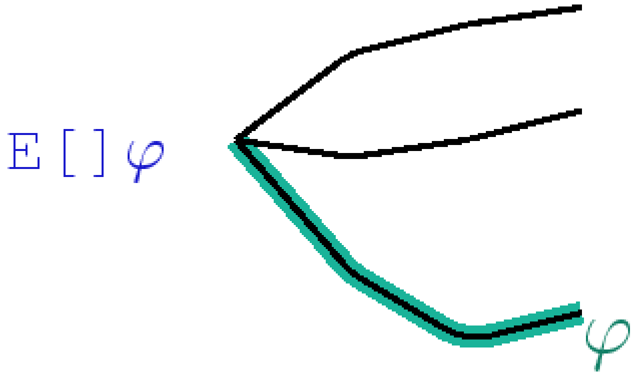

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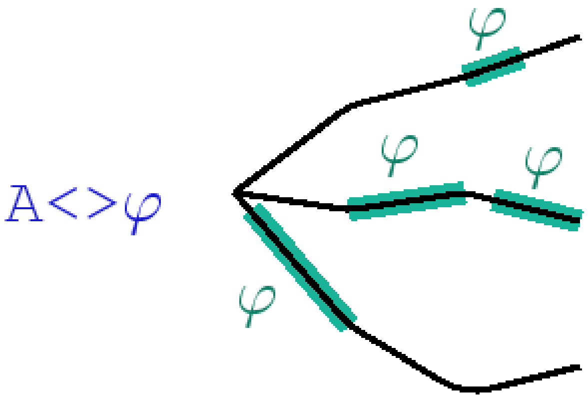

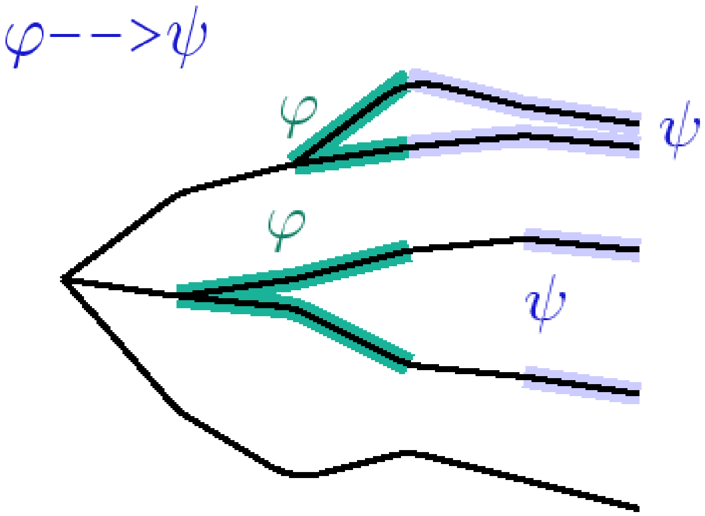

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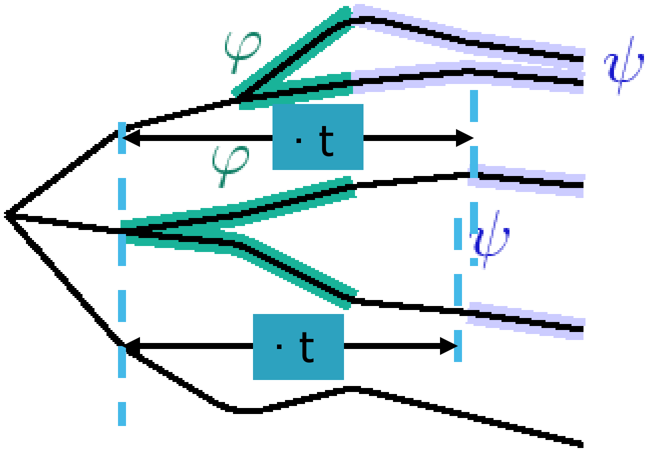

Jug Example

- Safety: Never overflow.
  - A[] forall(i:id_t) level[i] <= capa[i]
- Validation/Reachability: How to get 1 unit.
  - E<> exists(i:id_t) level[i] == 1

Train-Gate Crossing

- Safety: One train crossing.
  - A[] forall (i : id_t) forall (j : id_t) Train(i).Cross && Train(j).Cross imply i == j
- Liveness: Approaching trains eventually cross.
  - Train(0).Appr --> Train(0).Cross
  - Train(1).Appr --> Train(1).Cross
  - …
- No deadlock.
  - A[] not deadlock