Exercise 1. Prove that for all $n \in \mathbb{N}$

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad .$$

Exercise 2. Prove that for all $n \in \mathbb{N}$

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \quad .$$

Exercise 3. Prove that $n! > 2^n$ for $n \geqslant 4$.

Exercise 4. Prove that for all $n \in \mathbb{N}$,

$$x + 4x + 7x + \ldots + (3n-2)x = \frac{n(3n-1)x}{2} \quad .$$

Exercise 5. Prove that for all $n \in \mathbb{N}$, $10^{n+1} + 10^n + 1$ is divisible by 3.

Exercise 6. Prove that for all $n \in N$, $4 \cdot 10^{2n} + 9 \cdot 10^{2n-1} + 5$ is divisible by 99.

Exercise 7. Prove that for all $n \in \mathbb{N}$

$$1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1 \quad .$$

Exercise 8. Prove that for all $n \in \mathbb{N}$

$$\frac{1}{2} + \frac{1}{6} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1} \quad .$$

Exercise 9. Prove that $2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 2$ for all $n \geqslant 1$.

Exercise 10. Prove that $1 + 2 + 3 + \ldots n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$.

Exercise 11. Prove that for all $n \in \mathbb{N}, n \geqslant 1$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1} \quad .$$