## Formal methods Exercises 1

1) Given a transition system $\mathrm{M}=\left(S, S_{0}, L, R\right)$ (in the figure),
a) complete the specification of $M$ by substituting "..." with right symbols from figure;
b) draw the computation tree of M up to 4 levels starting from $\mathrm{s}_{0}$.


$$
\begin{aligned}
& S=\{\mathrm{s} 0, \ldots, \mathrm{~s} 3\} \\
& S_{0}=\{\mathrm{s} 0\} \\
& L: \quad l(\mathrm{~s} 0)=\{\mathrm{r}\} \text {, } \\
& l(\mathrm{~s} 1)=\{\mathrm{p}, \mathrm{t}, \mathrm{r}\} \\
& l(\mathrm{~s} 2)=\{\ldots\} \\
& l(\mathrm{~s} 3)=\{\ldots\} \\
& R=\{\ldots,\langle\mathrm{s} 2, \mathrm{~s} 0>, \ldots . .\}
\end{aligned}
$$

2) Specify the transition relation $R$ of model M (in figure) in symbolic form.

## Example

Transition <s2, s0> symbolically $R_{2,0} \equiv \neg \mathrm{p} \wedge \mathrm{q} \wedge \neg \mathrm{r} \wedge \neg \mathrm{t} \wedge \neg \mathrm{p}^{\prime} \wedge \neg \mathrm{q}^{\prime} \wedge \mathrm{r}^{\prime} \wedge \mathrm{t}^{\prime}$
3) a) Check the satisfiability of following CTL formulas for the transition system $M$ (see figure)
b) transform the formulas to base form (using EX, EG, EU and negation).
a) $\mathrm{M}, \mathrm{s}_{0}=\mathrm{EF}(\mathrm{q})$
b) $\mathrm{M}, \mathrm{s}_{0} \mid=\mathrm{EG}(\mathrm{r})$
c) $\mathrm{M}, \mathrm{s}_{2}=\mathrm{AG}(\mathrm{r})$
d) $\mathrm{M}, \mathrm{s}_{2} \mid=\neg \mathrm{EX}(\mathrm{r})$
e) $\mathrm{M}, \mathrm{s}_{0} \mid=\mathrm{A}((\mathrm{t} \vee \mathrm{p}) \mathrm{Uq})$
f) $\mathrm{M}, \mathrm{s}_{0}=\mathrm{E}(\mathrm{r}-->(\mathrm{t} \wedge \neg \mathrm{q}))$

Where --> denotes "leads to" operator not implication!
3. Given a symbolic state: $\varphi \equiv \neg \mathrm{x}_{1} \wedge \neg \mathrm{x}_{2}$ and transition relation $R \equiv \mathrm{x}_{0} \wedge\left(\mathrm{x}_{1} \Rightarrow \mathrm{x}_{2}\right) \wedge \neg \mathrm{x}_{0}{ }^{\prime} \wedge \neg \mathrm{x}_{1}{ }^{\prime} \wedge \neg \mathrm{x}_{2}{ }^{\prime}$,
find symbolic pre-image $\mathrm{EX}(\varphi) \equiv \exists V^{\prime}\left(R \wedge \varphi\left[V^{\prime} / V\right]\right)$ by showing separately $\left[V^{\prime} / V\right]-$ substitution and $\exists$-quantifier elimination.

