1. Show that the set of all finite bitstrings $\{0,1\}^*$ is countable.
2. Describe a Turing machine that computes function $y = 2x + 1$.
3. Show that $3n^2 + 6n + 7 = O(n^2)$.
4. Show that $2n^3 + 6n^2 + 6n + 1 = O(n^3)$.
5. Show that $n^3 \neq O(n^2)$.
6. Show that $n! \neq O(2^n)$.
7. Find functions $f(n)$ and $g(n)$ such that $f(n) = O(g(n))$, $g(n) \neq O(f(n))$, and $f(n) \neq o(g(n))$.
8. Given a list of functions in asymptotic notation, order them by growth rate (slowest to fastest).
9. Show that

$$\begin{array}{llllllllll} (a) & \Theta(n \log_2 n) & (b) & \Theta(n^2) & (c) & \Theta(n) & (d) & \Theta(1) & (e) & \Theta(2^n) \\ (f) & \Theta(n^3) & (g) & \Theta(n!) & (h) & \Theta(\log_2 n) & (i) & \Theta(n^2 \log_2 n) & (j) & \Theta(2^n \log^2 n) \end{array}$$

10. Check if the following conditions are true

$$\begin{array}{ll} (a) & \Theta(n+30) = \Theta(3n-1)\ , \\ (b) & \Theta(n^2+2n-10) = \Theta(n^2+3n)\ , \\ (c) & \Theta(n^3 \cdot 3n) = \Theta(n^2+3n)\ . \end{array}$$

11. Write each of the following functions in O notation.

$$(a)\quad 5 + 0.001n^3 + 0.025n \qquad (b)\quad 500n + 100n^{1.5} \qquad (c)\quad 0.3n + 5n^{1.5} + 2.5n^{1.75}$$

12. Show that