

1. Apply the Euclidean algorithm and calculate

$$\gcd(26, 9)$$

$$\gcd(81, 18)$$

2. Express the following pairs of numbers in the form of Bezout identity

$$\alpha a + \beta b = \gcd(a, b) .$$

$$(60, 12)$$

$$(12, 18)$$

$$(26, 9)$$

3. Find multiplicative modular inverse

$$2^{-1} \text{ in } \mathbb{Z}_7$$

$$4^{-1} \text{ in } \mathbb{Z}_{11}$$

$$9^{-1} \text{ in } \mathbb{Z}_{26}$$

$$2^{-1} \text{ in } \mathbb{Z}_6$$

4. Find additive inverse

$$-3 \text{ in } \mathbb{Z}_5$$

$$-4 \text{ in } \mathbb{Z}_{10}$$

5. How many invertible elements?

$$\mathbb{Z}_6$$

$$\mathbb{Z}_6^\times$$

$$\mathbb{Z}_{11}^\times$$

6. Which elements have multiplicative inverses in \mathbb{Z}_8 and \mathbb{Z}_{20} ?

7. Write out addition and multiplication tables in \mathbb{Z}_5 and \mathbb{Z}_8 .

8. Which numbers are invertible in \mathbb{Z}_8 and \mathbb{Z}_{20} ?

9. Solve the following linear equations

$$2x \equiv 1 \pmod{7}$$

$$4x \equiv 1 \pmod{11}$$

$$9x \equiv 1 \pmod{26}$$

$$x + 3 \equiv 2 \pmod{5}$$

$$5 + 6 \equiv x \pmod{11}$$

$$5x + 2 \equiv 3 \pmod{7}$$

$$4x + 3 \equiv 11 \pmod{12}$$

$$x - 4 \equiv 7 \pmod{12}$$

$$4x \equiv 2 \pmod{19}$$

$$4x + 3 \equiv 5 \pmod{13}$$

$$2x + 1 \equiv 9x - 4 \pmod{23}$$

$$5x - 1 \equiv 3x + 1 \pmod{26}$$

10. Solve the systems of linear equations

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 2a + b \equiv 0 \pmod{26} \end{cases}$$

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 4a + b \equiv 1 \pmod{26} \end{cases}$$

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 3a + b \equiv 0 \pmod{26} \end{cases}$$

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 16a + b \equiv 10 \pmod{26} \end{cases}$$

$$\begin{cases} 8a + b \equiv 8 \pmod{26} \\ 5a + b \equiv 13 \pmod{26} \end{cases}$$