1. Apply the Euclidean algorithm and calculate

$$\gcd(26, 9) \qquad\qquad \gcd(81, 18)$$

2. Express the following pairs of numbers in the form of Bezout identity

$$\alpha a + \beta b = \gcd(a, b) \ .$$

$$(60, 12) \qquad\qquad (12, 18) \qquad\qquad (26, 9)$$

3. Find multiplicative modular inverse

$$2^{-1} \text{ in } \mathbb{Z}_7 \qquad\qquad 4^{-1} \text{ in } \mathbb{Z}_{11}$$
$$9^{-1} \text{ in } \mathbb{Z}_{26} \qquad\qquad 2^{-1} \text{ in } \mathbb{Z}_6$$

4. Find additive inverse

$$-3 \text{ in } \mathbb{Z}_5 \qquad\qquad -4 \text{ in } \mathbb{Z}_{10}$$

5. How many invertible elements?

$$\mathbb{Z}_6 \qquad\qquad \mathbb{Z}_6^\times \qquad\qquad \mathbb{Z}_{11}^\times$$

6. Which elements have multiplicative inverses in $\mathbb{Z}_8$ and $\mathbb{Z}_{20}$?
7. Write out addition and multiplication tables in $\mathbb{Z}_5$ and $\mathbb{Z}_8$.
8. Which numbers are invertible in $\mathbb{Z}_8$ and $\mathbb{Z}_{20}$?
9. Solve the following linear equations

$$x + 3 \equiv 2 \pmod 5 \qquad 5 + 6 \equiv x \pmod{11} \qquad 5x + 2 \equiv 3 \pmod 7$$
$$4x + 3 \equiv 11 \pmod{12} \qquad x - 4 \equiv 7 \pmod{12} \qquad 4x \equiv 2 \pmod{19}$$
$$4x + 3 \equiv 5 \pmod{13} \qquad 2x + 1 \equiv 9x - 4 \pmod{23} \qquad 5x - 1 \equiv 3x + 1 \pmod{26}$$

10. Solve the systems of linear equations

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 2a + b \equiv 0 \pmod{26} \end{cases} \qquad \begin{cases} a + b \equiv 17 \pmod{26} \\ 4a + b \equiv 1 \pmod{26} \end{cases}$$

$$\begin{cases} a + b \equiv 17 \pmod{26} \\ 3a + b \equiv 0 \pmod{26} \end{cases} \qquad \begin{cases} 5a + b \equiv 21 \pmod{26} \\ 16a + b \equiv 10 \pmod{26} \end{cases}$$

$$\begin{cases} 8a + b \equiv 8 \pmod{26} \\ 5a + b \equiv 13 \pmod{26} \end{cases}$$