Data Mining, Lecture 8

Association Patten Mining

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What Pattern is?

- Pattern recognition is the discipline whose goal is the classification of objects into a number of classes or categories. [S.Theodoridis]
- What Pattern is? Object? Sub set?

Market basket data

- Most popular example is *Supermarket data*. The goal is to determine *associations* between groups of items bought by customers.
- Discovered sets of items are referred to as *large itemsets, frequent itemsets, or frequent patterns*.
- Main applications include supermarket data (or shopping basket data in general), text mining, generalization to dependency-oriented data types.
- Within this chapter initial data will be refereed as *transactions* and outputs as *itemsets*.

The Frequent Pattern Mining Model

- Let U be the d - dimensional universe of elements (goods offered by the supermarket) and $\mathcal{T}$ is the set of transactions $T_1, \dots, T_n$. They said that transaction T_i is drawn on universe of items U.
- T_i may be represented by d-dimensional binary record.
- *itemset* is the set of items. k-*itemset* is the itemset containing exactly k-items.

The Frequent Pattern Mining Model

Definition

Support *The support of an itemset I is defined as the fraction of the transactions in the database $\mathcal{T} = \{T_1, \dots T_n\}$ that contain I as the subset*

The support of the itemset I is defined by $sup(I)$. Not to be confused with supremum.

Definition

Frequent Itemset Mining Given a set of trasactions $\mathcal{T} = \{T_1, \dots T_n\}$ where each transaction T_i is drawn on the universe of elements U, determine all itemsets I that occure as a subset of at least a predefined fraction minsup of the transactions in $\mathcal{T}$.

Predefined fraction minsup is referred as *minimal support*.

Example: Market basket data set

tid	Set of items	Biary representation
1	{ Bread,Butter, Milk }	110010
2	{ Eggs, Milk, Yogurt }	000111
3	{ Bread, Cheese, Eggs, Milk }	101110
4	{ Eggs, Milk, Yogurt }	000111
5	{ Cheese, Milk, Yogurt }	001011

The Frequent Pattern Mining Mode

Definition

Frequent Itemset Mining: Set-wise Given as set of sets $\mathcal{T} = \{T_1, \dots T_n\}$, *where each transaction* T_i *is drawn on the universe of elements* U, *determine all sets* I *that occur as the subset of at least a predefined fractonminsup of the sets in* $\mathcal{T}$.

Support Monotonicity Property *The support of every subset* J *of* I *is at least equal to the of the support of itemset* I.

$$sup(J) \geq sup(I) \quad \forall J \subset I$$

Downward Closure Property *Every subset of the frequent itemset is also frequent.*

Definition

Maximal Frequent Itemsets *A frequent itemset is maximala at a given minimum support level minsup, if it is frequent and no superset of its frequent.*

Association Rule Generation Framework

Informal definition If the presence of item set X in the certain transaction(s) leads (implies) presence of the set of items Y in the same transaction(s) then we talk about rule $(X \Rightarrow Y)$.

Definition

Confidence *Let X and Y be two sets of items. The confidence of the rule $\mathrm{conf}(X \Rightarrow Y)$ conditional probability of $X \cup Y$ occurring in a transaction, given that the transaction contains X*

$$\mathrm{conf}\big(X \Rightarrow Y\big) = \frac{sup\big(X \cup Y\big)}{sup\big(X\big)}$$

Definition

Association Rule *Let X and Y be two sets of items. Then, the rule $X \Rightarrow Y$ is said to be an association rule at a minimum support of $minsup$ and minimum confidence $\min\mathrm{conf}$ if it satisfies following conditions.*

1. $sup\big(X \cup Y\big) \geq \min sup$
2. $\mathrm{conf}\big(X \Rightarrow Y\big) \geq minconf$

Frequent Itemset Mining Algorithms

- Brute force algorithms.
- The Apriori algorithm.
- Enumeration-Tree Algorithms
- Recursive Suffix-Based Pattern Growth Methods

The Apriori Algorithm

begin
$k = 1$;
$\mathcal{F}_1 = \{$ All Frequent 1-itemsets $\}$;
while $\mathcal{F}_k \neq \emptyset$
Generate $\mathcal{C}_{k+1}$ by joining itemset-pairs in $\mathcal{F}_k$;
Prune itemsets from $\mathcal{C}_{k+1}$ that violate downward closure;
Determine $\mathcal{F}_{k+1}$ by support counting on $\left(\mathcal{C}_{k+1}, T\right)$ and retaining from $\mathcal{C}_{k+1}$ with support of at least minsup;
$k = k + 1$;
end
return $\left(\cup_{i=1}^{k} \mathcal{F}_i\right)$
end

Alternative Models: Interesting Patterns

- Collective strength
- Statistical Coefficient of Correlation
- χ^2 Measure
- Nonlinear relationships

Collective strength

- An itenset is said to be in *violation* of transaction, if some of the items are present in the transaction and others are not.
- The *violation rate* $v(I)$ of the itemset I is defined as the fraction of violations of the itemset I over all transactions.
- The collective strength $C(I)$ of the itemset I is defined as follows

$$C(I) = \frac{1 - v(I)}{1 - E[v(I)]} \cdot \frac{R[v(I)]}{v(I)}.$$

- The expected value of the $v(I)$

$$R[v(I)] = 1 - \prod_{i \in I} p_i - \prod_{i \in I} (1 - p_i)$$

where p_i is the fraction of transactions where the item i occurs.

Collective strength

- Let us consider *violation* to be an unfavorable event (prospective of establishing a high correlation among items)
- Collective strength may be expressed as follows:

$$C(I) = \frac{\text{Good} \quad \text{events}}{E[\text{Good} \quad \text{events}]} \frac{E[\text{Bad} \quad \text{events}]}{\text{Bad} \quad \text{events}]}$$

- This leads us to the idea of *Negative Pattern Mining*. Determine patterns between the items or their absence.

Statistical Coefficient of Correlation

Covariance is the measure of the strength of correlation between two sets of random variables.

$$cov(X,Y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}$$

Correlation coefficient is standardized

$$\rho_{XY} = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$$

or in another form

$$\rho = \frac{E[XY] - E[X]E[Y]}{\sigma(X)\sigma(Y)}$$

Statistical Coefficient of Correlation

The Pearson correlation coefficient

$$\rho = \frac{E[XY] - E[X]E[Y]}{\sigma(X)\sigma(Y)}$$

May be rewritten in terms of *support* as follows

$$\rho_{ij} = \frac{sup(\{i,j\}) - sum(i) \cdot sup(j)}{\sqrt{sup(i) \cdot sup(j) \cdot (1 - sup(i)) \cdot (1 - sup(j))}}$$

Should we talk here about regression?

χ^2 measure

χ^2 test allows to assess if unpaired observations of two categorical variables are independent of each other or not.

$$\chi^2 = \sum_{i=1}^{\nu_1 \cdot \nu_2} \frac{\left(\mathcal{O}_i - E_i\right)^2}{E_i}$$

where ν_1 and ν_2 are the degrees of freedom (number of categories) in the first and in second variables respectively. In the case of binary data $\nu_1 \cdot \nu_2 = 2^{|X|}$.

Nonlinear

- $$y(x) = a_1 x^n + a_2 x^{n-1} + \ldots + a_n x + b$$

- $$y(x) = f(x)$$

  where $f(\cdot)$ is arbitrary nonlinear function