

## Homework 1 – Sets

**Exercise 1.** Let  $\mathbb{R}$  be the universal set. Given the two sets

$$A = \{x \in \mathbb{R} : 0 < x \leq 3\} ,$$
$$B = \{x \in \mathbb{R} : 2 \leq x < 4\} ,$$

define the following sets:  $A \cap B$ ;  $A \cup B$ ;  $A \setminus B$ ;  $A'$ .

**Exercise 2.** Suppose that

$$A = \{x \in \mathbb{N} : x \text{ is even}\} ,$$
$$B = \{x \in \mathbb{N} : x \text{ is prime}\} ,$$
$$C = \{x \in \mathbb{N} : x \text{ is a multiple of } 5\} .$$

Describe each of the following sets.

$$(a) \quad A \cap B$$
$$(b) \quad B \cap C$$
$$(c) \quad A \cup B$$
$$(d) \quad A \cap (B \cup C)$$

**Exercise 3.** If  $A = \{a, b, c\}$ ,  $B = \{1, 2, 3\}$ ,  $C = \{x\}$ , and  $D = \emptyset$ , list all of the elements in each of the following sets.

$$(a) \quad A \times B$$
$$(b) \quad B \times A$$
$$(c) \quad A \times B \times C$$
$$(d) \quad A \times D$$

**Exercise 4.** Let  $A$  be a set. Show that  $A \cap A = A$ .

**Exercise 5.** Let  $A$  be a set. Show that  $A \cup \emptyset = A$ .

**Exercise 6.** Let  $A, B, C$  be sets. Show that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

**Exercise 7.** Let  $A, B$  be sets. Show that  $A \cup B = B \cup A$ .

**Exercise 8.** Let  $A, B, C$  be sets. Show that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

**Exercise 9.** Let  $A, B$  be sets. Show that  $(A \cup B)' = A' \cap B'$ .

**Exercise 10.** Let  $A, B, C$  be sets. Show that  $A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)$ .